Assignment 8 STP 526, Fall 2009, S. Lohr

Due on Wednesday, 4 December 2009

1. Do problem 4.16 (a) on p. 135.

2. Suppose $Y_1, \ldots, Y_n$ are independent with $Y_i \sim N_1(x_i\beta, \sigma^2 x_i^2)$ and $x_i > 0$.

   (a) What is the generalized least squares estimator of β, $\hat{\beta}_{GLS}$?

   (b) What is $V(\hat{\beta}_{GLS})$? How does this variance compare to the variance of the estimator $\hat{\beta}_{OLS}$, where $\hat{\beta}_{OLS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$?

3. Consider the balanced randomized complete block design (RCBD) model

$$Y_{ij} = \tau_i + v_j + \epsilon_{ij},\ i = 1, \ldots, a;\ \ j = 1, \ldots, b$$

   where $\tau_1, \ldots, \tau_a$ are treatment means and $v_1, \ldots, v_b$ are block effects. We assume that $v_j \sim N(0, \sigma_B^2)$, $\epsilon_{ij} \sim N(0, \sigma^2)$ and all v_j's and ϵ_{ij}'s are independent.

   (a) For $a = 3$ and $b = 4$, write the model in the form

$$\mathbf{Y} = \mathbf{X}\tau + \mathbf{Z}\mathbf{v} + \epsilon,$$

   where τ is a vector of fixed effects and $\mathbf{v}$ is a vector of random effects. Note that there is no intercept term in the formulation above so that the $\mathbf{X}$ matrix will be full rank.

   (b) Find $\text{Cov}\,(\mathbf{Y})$.

   (c) Find the generalized least squares estimator of the vector τ.

   (d) Find the ordinary least squares estimator of τ. Is this the same as the generalized least squares estimator?

   (e) Find $\text{Cov}\,(\hat{\tau}_{GLS})$.