Assignment 5 — STP 526, Fall 2009, S. Lohr

Due on Monday, 19 October 2009

1. Read Sections 5.1, 5.2, and 5.4 of the text. Also read pages 164–168 of Section 5.3 (about the non-central chi-squared distribution).

2. (a) Find $\int\int\int \exp[-(2x^2 + 2y^2 + 4z^2 + 2xy + 2xz - 10x + 2y - 14z + 24)]\,dx\,dy\,dz$.

   (b) Find $\int\int\int (x^2+y^2)\exp[-(2x^2+2y^2+4z^2+2xy+2xz-10x+2y-14z+24)]\,dx\,dy\,dz$.

   Find these by using the multivariate normal density and expected value of quadratic forms. Do not use Result 5.1.3.

3. Do problems 5.19, 5.28(b,c) and 5.29 in the book. For problem 5.28, assume $\sigma^2 = 1$.

4. Let $Y_1, Y_2, \ldots, Y_n$ be independent $N(\mu, \sigma^2)$ random variables. Let $W_i = Y_i - Y_{i-1}$ for $i = 2, \ldots, n$ and let $\mathbf{W} = [W_2, W_3, \ldots, W_n]'$.

   (a) What is the distribution of $\mathbf{W}$?

   (b) Let $U = Y_1 + Y_2 + Y_3$. Are U and $\mathbf{W}$ independent if $n = 3$? If $n > 3$? Why, or why not?

5. In general, multivariate normal probabilities need to be calculated numerically. There are many algorithms for doing this. One of the simplest is to generate a large number of random vectors from the distribution and then calculate the proportion of the vectors that fall in the region of interest.

   The R function *mvrnorm* in the MASS package generates observations from a multivariate normal distribution.

   Using mvrnorm, write your own R function to calculate $P(a_1 \le Y_1 \le b_1, a_2 \le Y_2 \le b_2)$ for constants a_1, a_2, b_1, and b_2, where $\mathbf{Y} \sim N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Generate a very large number of realizations from the distribution of $\mathbf{Y}$, and then calculate the proportion of those that fall in the interval as your estimate of the probability.

   Apply your function to the following problems:

   (a) $\boldsymbol{\mu} = \mathbf{0}$, $\boldsymbol{\Sigma} = \mathbf{I}$, $a_1 = a_2 = 0$, $b_1 = b_2 = \infty$ (Use this to check your program, since we know what the answer to this one should be.)

   (b) $\boldsymbol{\mu} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\boldsymbol{\Sigma} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $a_1 = a_2 = 0$, $b_1 = b_2 = 5$

   (c) $\boldsymbol{\mu} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\boldsymbol{\Sigma} = \begin{bmatrix} 2 & 1.98 \\ 1.98 & 2 \end{bmatrix}$, $a_1 = a_2 = 0$, $b_1 = b_2 = 5$

   For this problem, turn in your code and the output for the problems. Please do not turn in a listing of all the random numbers you generate!

Suggested additional problems (DO NOT HAND IN): 5.8(a), 5.9, 5.10, 5.11, 5.13, 5.21, 5.26, 5.27, 5.30