Assignment 4 STP 526, Fall 2009, S. Lohr

Due on Wednesday, 7 October 2009

1. Read Sections 4.1-4.4 of the text.

2. (Two-way unbalanced ANOVA without interaction) Consider the model

$$Y_{ijk} = \mu + \tau_i + \beta_j + \varepsilon_{ijk}, \tag{1}$$

for $i = 1, 2$; $j = 1, 2, 3$; and $k = 1, \ldots, n_{ij}$. The within-cell sample sizes n_{ij} are given in the following table:

$i \backslash j$	1	2	3
1	3	2	0
2	1	3	2

Let

$$\boldsymbol{\beta} = [\mu\, \tau_1\, \tau_2\, \beta_1\, \beta_2\, \beta_3]^T.$$

   (a) Write the model as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \varepsilon$. Give $\mathbf{X}$ explicitly. What is rank($\mathbf{X}$)?

   (b) Find $\mathbf{X}'\mathbf{X}$ and a g-inverse $(\mathbf{X}'\mathbf{X})^-$.

   (c) Find $\mathbf{P_X}$, the perpendicular projection matrix onto $\mathcal{C}(\mathbf{X})$.

   (d) Give the set of all least squares estimators $\hat{\boldsymbol{\beta}}$.

   (e) What functions $\mathbf{c}'\boldsymbol{\beta}$ are estimable?

   (f) Which of the following are estimable: μ? $\beta_1 - \beta_2$? $\tau_1 + \beta_3$? For each of these functions that is estimable, find its BLUE.

3. Let $Y_1, \ldots, Y_n$ be uncorrelated with $E[Y_i] = \mu$ and $\text{Var}\,[Y_i] = \sigma^2$.
Let $S = \sum_{i=2}^n (Y_i - Y_{i-1})^2$.

   (a) Write S as a quadratic form $\mathbf{Y}'\mathbf{AY}$.

   (b) Using matrices, find $E[S]$ and give an unbiased estimator of σ^2 that is a linear function of S.

4. Do problems 4.6, 4.7, and 4.8 (for 4.8, also give conditions for which β_0 will be estimable, and give conditions under which β_1 will be estimable) in the book.

Suggested additional problems (DO NOT HAND IN): 4.2, 4.4, 4.13, 4.14