Assignment 2 STP 526, Fall 2009, S. Lohr

Due on Wednesday, September 9, 2009

1. Read Sections 2.1-2.6 of the book.

2. (a) Find the eigenvalues and eigenvectors (give the normalized eigenvectors) of

$$\mathbf{A} = \begin{bmatrix} 5 & -1 \\ -1 & 2 \end{bmatrix}$$

   (For this problem, calculate the eigenvalues and eigenvectors by hand, then check your result using R.)

   (b) Find an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}'\mathbf{A}\mathbf{P} = \mathbf{D}$.

   (c) Plot the eigenvectors (either by hand or using R).

3. Determine whether each of the following matrices is (i) orthogonal, (ii) nonnegative definite, (iii) positive definite, (iv) idempotent. You may use R for your matrix calculations (cut and paste the R code and output into your homework; bald, unannotated R output is unacceptable).

$$(a) \begin{bmatrix} 150 & 0 & -270 \\ -545 & 625 & -494 \\ 1125 & 0 & 1725 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} & 0 \\ 1/\sqrt{12} & 1/\sqrt{12} & 1/\sqrt{12} & -3/\sqrt{12} \end{bmatrix}$$

$$(c) \begin{bmatrix} 4 & 2 & 6 \\ 2 & 5 & -5 \\ 6 & -5 & 27 \end{bmatrix}$$

$$(d) \begin{bmatrix} 2 & 1 & 6 \\ 1 & 5 & 0 \\ 6 & 0 & 20 \end{bmatrix}$$

$$(e)\quad \mathbf{A}(\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}', \text{ where } \mathbf{A} = \begin{bmatrix} 1 & 5 & 0 \\ 0 & -2 & 1 \\ 3 & 0 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

4. For the matrices in the previous problem that are nonnegative definite (including those that are positive definite), find (a) a matrix $\mathbf{B}$ such that $\mathbf{A} = \mathbf{B}\mathbf{B}'$, and (b) the square root of $\mathbf{A}$.

5. Prove results 11.16 and 11.17 (p. 11) in the matrix handout.

6. Do problems 2.16 and 2.26 in the text.

Suggested additional problems (DO NOT HAND IN): 2.7, 2.9, 2.12, 2.14, 2.17, 2.19, 2.20 (hint: look at the variance of the nonzero eigenvalues), 2.24, 2.25.
Use the Gram-Schmidt theorem (Result 1.2.4) to construct an orthogonal 4×4 matrix whose first column is

$$\begin{bmatrix} 5/7 \\ 2/7 \\ 2/7 \\ 4/7 \end{bmatrix}$$

For advanced R users: try writing an R function to do the Gram-Schmidt orthogonalization.