

MAT576 final homework set

final version

Due Monday 5 pm in my mailbox or by email

Do the following problems from the textbook:

p142/5.24

p162/5.51 (*hint: use spherical coordinates such that θ becomes the angle between \mathbf{r} and ξ*)

p172/5.58

Additional Problems

1. A couple of technical exercises:

- (a) There is no general product rule for distributional derivatives since products of distributions are generally not defined. But if ψ is an infinitely differentiable function and F is a distribution then we define ψF by

$$\langle \psi F, \phi \rangle = \langle F, \psi \phi \rangle.$$

Does the product rule for derivatives hold in this case? Explain as fully as you can.

- (b) Let f and g be two distributions of compact support in \mathbb{R}^n and let \mathfrak{F} denote the Fourier transform. Prove that

$$\mathfrak{F}(f * g) = (2\pi)^{n/2} \mathfrak{F}(f) \mathfrak{F}(g).$$

In particular you need to justify that in this case the multiplication of the distributions $\mathfrak{F}(f)$ and $\mathfrak{F}(g)$ makes sense.

2. In \mathbf{R}^3 define F to be the distribution whose value at the test function ϕ is the average value of ϕ on the unit sphere centered at the origin:

$$\langle F, \phi \rangle = \int_{|x|=1} \phi(x) dS.$$

Find a solution of $\Delta u = F$. *Hint: This problem can be done either by solving the radially symmetric equation directly or it can be done using Fourier transforms.*