

## Extra problem set 6

1. In  $\mathbf{R}^3$  define  $F$  to be the distribution whose value at the test function  $\phi$  is the average value of  $\phi$  on the unit sphere centered at the origin. Find a solution of  $\Delta u = F$ .

2. Define  $G \in \mathcal{D}(\mathbf{R})'$  by

$$\langle G, \phi \rangle = \lim_{\epsilon \rightarrow 0^+} \int_{|x| > \epsilon} \frac{\phi(x)}{x} dx.$$

Show that  $G$  is in fact a distribution.

3. There is no general product rule for distributional derivatives since products of distributions are generally not defined. But if  $\psi$  is an infinitely differentiable function and  $F$  is a distribution then we define  $\psi F$  by

$$\langle \psi F, \phi \rangle = \langle F, \psi \phi \rangle.$$

Does the product rule for derivatives hold in this case? Explain as fully as you can.