MAT505 - Final Problem Set

I strongly recommend that you use a CAS. If you need help with Maple please come and see me. To do the plots you can merely edit some of the Maple worksheets that are available at the MAT505 web link.

1. Consider the asymptotic matching problem for the equation

$$\epsilon\, y'' + \sqrt{x}\, y' - \sqrt{x}\, y = 0$$

   with boundary values

$$y(0) = A, \quad y(1) = B.$$

   (a) Find the thickness of the boundary layer.

   (b) The asymptotic series for the outer solution is $\sum \epsilon^n y_n(x)$. Find the recursion formula for the y_n.

   (c) The asymptotic series for the inner solution is $\sum \eta^n Y_n(z)$. What is η? Find the recursion formula for the Y_n.

   (d) Find the leading order uniform solution.

   (e) For $A = 0$, $B = 2$, and for each $\epsilon = 0.1, 0.01, 0.001$ plot the leading order uniform solution and the numerical solution together on the same graph.

2. Do problem 9.39a in the text, using asymptotic matching to solve the boundary-layer problem

$$\epsilon y'' - y' + \exp(y) = 0,\ y(0) = A < 0,\ y(1) = 0.$$

   Plot to compare the leading order uniform solution to the numerically computed solution for $\epsilon = 0.2$ with $A = -1,\ -0.1,\ -0.01$.

3. Use WKB to obtain an approximation for the large eigenvalues, and corresponding eigenfunctions, of

$$4(x-1)y'' + 2y' + \lambda^2 xy = 0,$$

$$y(2) = 0, y'(5) = 0.$$

   For large n the n^{th} eigenvalue λ_n^2 satisfies

$$\lambda_n \approx \alpha + \beta n$$

   and the corresponding eigenfunction has the form

$$y_n(x) \approx \frac{\sin(\lambda_n Q(x))}{R(x)}.$$

   Your final answer should consist of:

   (a) The numbers α and β in fixed point format.

   (b) The functions Q and R.

   To get Q you will need to evaluate an integral that is doable by hand, but a CAS will make it much easier.

4. Do problem 11.11 in the text concerning a multiple scale solution for the Van der Pol oscillator:

$$y'' + y - \epsilon(1 - y^2)y' = 0.$$

   For each of the following cases plot on the same graph both the numerical solution and the multiple scale solution:

   (a) $\epsilon = 0.01,\ y(0) = 0,\ y'(0) = 20.$

   (b) $\epsilon = 0.01,\ y(0) = 0,\ y'(0) = 0.5.$

   (c) $\epsilon = 0.1,\ y(0) = 0,\ y'(0) = 20.$

   (d) $\epsilon = 0.01,\ y(0) = 0,\ y'(0) = 20.$