

MAT 476 - Spring 2004 - Extra problem set 1

1. Consider the problem

$$2u_x + xu_y = u^2$$

with initial data $u(0, y) = 1$ for $y \geq 0$ and $u(x, 0) = 1/(1+x)$ for $x \geq 0$.

- (a) Sketch several typical characteristics in the plane.
- (b) Find the solution.
- (c) Is the solution defined in the entire plane? Explain.

2. Consider the problem

$$u_x + 2uu_y = 1$$

with initial data $u(x, 1) = 1$ for $-\infty < x < \infty$.

- (a) Find the solution.
- (b) Sketch several typical characteristics in the plane.
- (c) Is the solution defined in the entire plane? Explain.
- (d) What kind of surface is the parameterized surface that you obtained?
- (e) What part of the parameterized surface represents the solution?
- (f) Why is the initial data $u(x, 0) = 0$ not appropriate for this problem?