Final Answer

(a) steady state solution $v := v^{(1)} + v^{(2)}$

$$v^{(1)}(x,y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{F_{mn}}{\lambda_{mn} k} \phi_{mn}(x,y)$$

where $\phi_{mn}(x,y) = \sin\frac{m\pi x}{a} \cos\frac{(2n-1)\pi y}{2b}$

$$F_{mn} = \frac{4}{ab}\int_0^b\int_0^a F(x,y)\,\phi_{mn}(x,y)\,dx\,dy$$

$$\lambda_{mn} = \frac{m^2\pi^2}{a^2} + \frac{(2n-1)^2\pi^2}{4b^2}$$

$$v^{(2)}(x,y) = \sum_{n=1}^{\infty} \alpha_n \sinh\frac{(2n-1)\pi x}{2b} \cos\frac{(2n-1)\pi y}{2b}$$

where $\alpha_n = \dfrac{2\int_0^b \phi(y)\cos\frac{(2n-1)\pi y}{2b}\,dy}{b\sinh\frac{(2n-1)\pi a}{2b}}$

(b)

$$w(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} c_{mn}\,\phi_{mn}(x,y)\,e^{-\lambda_{mn} k t}$$

where

$$c_{mn} = \frac{4}{ab}\int_0^b\int_0^a w_0(x,y)\,\phi_{mn}(x,y)\,dx\,dy$$

and $w_0(x,y) = u_0(x,y) - v^{(1)}(x,y) - v^{(2)}(x,y)$