

Homework Set 9

From the text:

4.6/3

Extra Problems:

1. Consider the problem

$$u_t = k(u_{xx} + u_{yy}) + F(x, y), \quad 0 < x < a, \quad 0 < y < b, \quad t > 0,$$

(k is a positive constant) with boundary conditions

$$u_y(x, 0, t) = 0, \quad u(x, b, t) = 0, \quad u(0, y, t) = 0, \quad u(a, y, t) = \phi(y),$$

and initial condition $u(x, y, 0) = u_0(x, y)$.

(a) Find the steady state solution (split the problem into 2 problems).

(b) Find the transient solution.

2. Consider the heat equation

$$u_t = ku_{xx},$$

in $\mathcal{D} := \{(x, t) : 0 < x < L, 0 < t < T\}$. The “parabolic boundary” is the set

$$\mathcal{P} := \{(x, 0) : 0 \leq x \leq L\} \cup \{(0, t) : 0 < t \leq T\} \cup \{(L, t) : 0 < t \leq T\}.$$

Give a physical argument that shows that

$$\sup_{\mathcal{D}} u(x, t) \leq \sup_{\mathcal{P}} u(x, t).$$