NAME.....

MAT 462
Take-home final exam
Spring 2006

Consider the heat equation in a rectangular plate:

$$u_t = k(u_{xx} + u_{yy}) + F(x, y), \quad 0 < x < a,\ 0 < y < b,\ t > 0,$$

with boundary conditions

$$u(x, 0, t) = 0,\ u(x, b, t) = f(x),\ u_x(0, y) = 0,\ u_x(a, y) = g(y),$$

and initial condition $u(x, y, 0) = u_0(x, y)$.

1. Find the steady-state solution $v(x, y)$.
   (*Hint: split the steady-state problem into 3 simpler problems*).

2. Find the solution u to the heat equation and show that it will approach the steady state as $t \to \infty$.

3. Let
   $$E(t) := \int_0^a \int_0^b u(x, y, t)\, dy\, dx.$$
   This represents the energy in the plate. Find an expression for dE/dt and try to identify the physical significance of each term.

4. Let
   $$J(t) := \int_0^a \int_0^b [u(x, y, t) - v(x, y)]^2\, dy\, dx.$$
   This quantity is useful for mathematical purposes, but does not seem to have any physical significance. Compute dJ/dt. Can you draw any conclusions?