This Maple worksheet shows how one may factor $\left(a^{\frac{p}{q}} - b^{\frac{p}{q}}\right)$ in order to show that the function $f(x) = x^{\frac{p}{q}}$ is continuous on the positive real line.

```
assume(p > 0);
assume(a > 0);
assume(q > 0);
assume(b > 0); assume(a > b);
assume(p :: integer);
assume(q :: integer);
```

$$simplify\left((a-b)\cdot\frac{sum\left(a^{\frac{i}{q}}\cdot b^{\frac{(p-1-i)}{q}}, i=0..(p-1)\right)}{sum\left(a^{\frac{i}{q}}\cdot b^{\frac{(q-1-i)}{q}}, i=0..(q-1)\right)}\right)$$

$$-\frac{-a\sim^{\left(\frac{p\sim+1}{q\sim}\right)} + a\sim^{\left(\frac{p\sim}{q\sim}\right)} b\sim^{\left(\frac{1}{q\sim}\right)} + b\sim^{\left(\frac{p\sim}{q\sim}\right)} a\sim^{\left(\frac{1}{q\sim}\right)} - b\sim^{\left(\frac{p\sim+1}{q\sim}\right)}}{a\sim^{\left(\frac{1}{q\sim}\right)} - b\sim^{\left(\frac{1}{q\sim}\right)}} \quad (1)$$

$A := denom(\%);$

$$a\sim^{\left(\frac{1}{q\sim}\right)} - b\sim^{\left(\frac{1}{q\sim}\right)} \quad (2)$$

$B := numer(\%\%);$

$$a\sim^{\left(\frac{p\sim+1}{q\sim}\right)} - a\sim^{\left(\frac{p\sim}{q\sim}\right)} b\sim^{\left(\frac{1}{q\sim}\right)} - b\sim^{\left(\frac{p\sim}{q\sim}\right)} a\sim^{\left(\frac{1}{q\sim}\right)} + b\sim^{\left(\frac{p\sim+1}{q\sim}\right)} \quad (3)$$

$expand\left(A\cdot\left(a^{\frac{p}{q}} - b^{\frac{p}{q}}\right)\right) - B;$

$$a\sim^{\left(\frac{p\sim}{q\sim}\right)} a\sim^{\left(\frac{1}{q\sim}\right)} + b\sim^{\left(\frac{p\sim}{q\sim}\right)} b\sim^{\left(\frac{1}{q\sim}\right)} - a\sim^{\left(\frac{p\sim+1}{q\sim}\right)} - b\sim^{\left(\frac{p\sim+1}{q\sim}\right)} \quad (4)$$

$simplify(\%);$

$$0 \quad (5)$$

This shows that $\left(a^{\frac{p}{q}} - b^{\frac{p}{q}}\right)$ may be written as $\dfrac{(a-b)\left(\sum_{i=0}^{p-1} a^{\left(\frac{i}{q}\right)} b^{\left(\frac{p-1-i}{q}\right)}\right)}{\sum_{i=0}^{q-1} a^{\left(\frac{i}{q}\right)} b^{\left(\frac{q-1-i}{q}\right)}}$