Review questions for chapter 0

1. Try to prove that
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\,.$$

2. Prove that
$$A \setminus (B \setminus C) = (A \cap C) \cup (A \setminus B)\,.$$

3. Is the composition of two injections an injection? Why?

4. Is the composition of two surjections a surjection? Why?

5. Let $|C|$ denote the cardinality of the set C. Let A and B be sets. Explain carefully what is meant by each of these: $|A| = |B|$, $|A| \leq |B|$, $|A| < |B|$.

6. Show that the interval $(0, 1)$ and $\mathbb{R}$ are cardinally equivalent.

7. Show that the interval $[0, 1]$ and $\mathbb{R}$ are cardinally equivalent.

8. Let $p \in \mathbb{N}$ and $z > 0$. Let
$$S := \{x \in \mathbb{R} \mid x^p \leq z, \quad x \geq 0\}.$$
   (a) Show that $S \neq \emptyset$.
   (b) Show that S is bounded from above by $z + 1$.
   (c) Let q be the least upper bound of S. It can be shown that if $q^p > z$ then for a sufficiently small number $\delta > 0$ we have that $q - \delta$ is an upper bound for S. It can also be shown that if $q^p < z$ then for a sufficiently small number $\delta > 0$ we have that $(q + \delta)^2 \leq z$. Use these facts to prove that if q is the least upper bound of S then $q^p = z$, i.e. $q = \sqrt[p]{z}$.
   (d) Let A and B be two finite sets, $|A| = m$ and $|B| = n$, and let $\mathfrak{F}(A, B)$ be the set of all functions from A to B. What is $|\mathfrak{F}(A, B)|$ equal to? What is the cardinality of the set $\mathfrak{F}(\mathbb{N}, \{0, 1\})$?

9. Suppose $a < x < b$. Show there is a number $\epsilon > 0$ such that $a < x - \epsilon < x < x + \epsilon < b$.