Basic Integration

1. $$\int u^p \, du = \frac{1}{p+1} u^{p+1} + C, \quad p \neq -1$$

2. $$\int \frac{1}{u} \, du = \ln |u| + C$$

3. $$\int \exp(cu) \, du = \exp(cu)/c + C$$

4. $$\int \sin(u) \, du = -\cos(u) + C$$

5. $$\int \cos(u) \, du = \sin(u) + C$$

6. $$\int \sec^2(u) \, du = \tan(u) + C$$

7. $$\int \csc^2(u) \, du = -\cot(u) + C$$

8. $$\int \sec(u) \, \tan(u) \, du = \sec(u) + C$$

9. $$\int \csc(u) \, \cot(u) \, du = -\csc(u) + C$$

10. $$\int \frac{du}{\sqrt{1-u^2}} = \arcsin(u) + C$$

11. $$\int \frac{du}{1+u^2} \, du = \arctan(u) + C$$

NOTE: You can interpret $u = u(x)$ and then replace du by $u'(x)\,dx$. For example

$$\int \frac{u'(x)}{u(x)} \, dx = \ln |u(x)| + C$$

This is a special case of the **method of substitution** that tells us that if F is an antiderivative of f then

$$\int f(u(x))\, u'(x) \, dx = F(u(x)) + C.$$

The $u'(x)$ **is necessary** so don't do things like

$$\int \frac{dx}{1+x^4} = \ln(1+x^4) + C.$$

What is true is that:

$$\int \frac{4x^3\,dx}{1+x^4}\,dx = \ln(1+x^4) + C.$$

Another method is **integration by parts**:

$$\int u(x)\,v'(x)\,dx = u(x)v(x) - \int u'(x)\,v(x)\,dx.$$

Finally, it will be useful for you to review the **method of partial fractions**. To integrate a rational function (with numerator of smaller degree than denominator) such as

$$\frac{\alpha+\beta x+\gamma x^2+\delta x^3+\epsilon x^4+\eta x^5+\kappa x^6+\lambda x^7+\mu x^8}{(x+a)(x+b)^2(x^2+px+q)(x^2+mx+n)^2}$$

we find constants A, B, etc. such that the above fraction can be written as a sum of simpler fractions:

$$\frac{A}{x+a}+\frac{B}{(x+b)^2}+\frac{C}{x+b}+\frac{Dx+E}{x^2+px+qx}+\frac{Ex+F}{(x^2+mx+n)^2}+\frac{Gx+H}{x^2+mx+n}.$$

Each if the above terms can now be integrated (the quadratics in the denominator may require completion of the square followed by substitution. For example

$$\int \frac{1}{x^2-4x+29}\,dx = \int \frac{1}{(x-2)^2+5^2}\,dx$$

$$= \int \frac{5du}{25(u^2+1)} = (1/5)\arctan(u) + C = (1/5)\arctan((x-2)/5) + C,$$

where $u := (x-2)/5$. You should do all the problems on the following page and check your answers with Maple. Once in a while the answer Maple gives may be different from your answer not because you are wrong but because the answers differ only in form. Maple does not include the arbitrary constant. **You should**. Suppose you get the answer $-\sin^2(x)+C$ and Maple says the answer is $\cos^2(x)$, then both are right. Can you see why?

Practice Problems

1. $$\int \frac{1}{\cos^2(x)}\,dx$$

2. $$\int \frac{2}{x^2+4}\,dx$$

3. $$\int \frac{2x+9}{x^2+2x+10}\,dx$$

4. $$\int \frac{x}{x^4+1}\,dx$$

5. $$\int \frac{x}{x^2+1}\,dx$$

6. $$\int (3x+7)^{20}\,dx$$

7. $$\int (3x+7)^{-1}\,dx$$

8. $$\int \frac{\cos(x)}{1+\sin^2(x)}\,dx$$

9. $$\int \frac{\cos(x)}{(1+\sin(x))^2}\,dx$$

10. $$\int x\,\exp(x^2)\,dx$$

11. $$\int e^x\,\sin^3(e^x)\,\cos(e^x)\,dx$$

12. $$\int x^{-1}\sec^2(\ln(x))\,dx$$

13. $$\int \tan(5x)\,\sec^2(5x)\,dx$$

14. $$\int \frac{\csc^2(3x)}{\sqrt{4-\cot^2(3x)}}\,dx$$

15. $$\int \exp(2\ln(\csc(x)))\,dx$$

16. $$\int \frac{2}{x^2+4}\,dx$$

17. $$\int \frac{2+x}{x^2-x-2}\,dx$$

18. $$\int \frac{5}{x^2-4x}\,dx$$

19. $$\int \frac{1+2x+3x^2+4x^3}{(5x^2+6)^2}\,dx$$

20. $$\int \frac{3+x}{(x^2+4)(x^2+1)}\,dx$$

21. $$\int x\sin(x)\,dx$$

22. $$\int x^2\ \arctan(x)\,dx$$

23. $$\int x\ \arcsin(x)\,dx$$

24. $$\int x\,e^{-ax}\,dx$$

25. $$\int \frac{3+x}{x(7x+1)^2}\,dx$$

26. $$\int \frac{x^4}{(x-1)(x-2)(x-3)}\,dx$$