NAME.....

MAT 274, summer 2009,
Final Exam
(100 points)

1. (15 pts) Find the general solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{A} = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix}$.

2. (5 pts) The linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ has general solution

$$c_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{t/2} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-6t}.$$

Sketch the phase portrait (some trajectories) in the phase plane.

3. (5 pts) For the equation

$$y' = y(4 - y^2)$$

(a) Sketch the phase line diagram.

(b) Find all equilibria and determine if they are stable or unstable.

4. (10 pts) Solve:

$$y' = e^{-y}(1+t),$$

satisfying the initial condition $y(0) = 1$.

5. (5 pts) Find the Laplace transform Y of the solution to $y'' + 7y' + 21y = 1$ with initial conditions $y(0) = 0,\ y'(0) = 1$.

6. (10 pts) Find the solution of

$$y' + y/t = 1, \;\; y(1) = -2.$$

7. (5 pts) Determine the maximal interval on which the following problem has a solution. **Do not solve!**

$$ty'' + \sqrt{5-t}\, y' + y = \frac{t}{t-6}, \qquad y(1) = -2.$$

8. (10 pts) Use Euler's method with step size $h = 0.1$ to find an approximate value of $y(0.3)$ if y is the solution of

$$y' + \frac{1}{y} = t^2$$

with initial condition $y(0) = 1$

9. (5 pts) Write the equation $y'' + 2(y')^2 - 2e^t y = \sin(t)$ as a first order system.

10. (10 pts) Find the undetermined coefficients **form** of the particular solution for the following problem. DO NOT COMPUTE THE VALUES OF THE COEFFICIENTS.

$$y'' + 2y' + 5y = e^{-t}\sin 2t + t^2 + 7\sin(2t).$$

11. (10 pts) Find the inverse Laplace transform:

$$\frac{3s+5}{s^2+4s+13},$$

12. (10 pts)Use **variation of parameters** to find a particular solution for

$$y'' - y = e^{2t}.$$

Table 1: **Laplace Transforms**

$f(t)$	$F(s)$
$f'(t)$	$sF(s) - f(0)$
$u_a(t)f(t-a)$	$\mathrm{e}^{-as}F(s)$
$\mathrm{e}^{at}f(t)$	$F(s-a)$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{at}\sin(\omega t)$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at}\cos(\omega t)$	$\frac{s-a}{(s-a)^2+\omega^2}$
$te^{at}\sin(\omega t)$	$\frac{2\omega(s-a)}{\{(s-a)^2+\omega^2\}^2}$
$te^{at}\cos(\omega t)$	$\frac{(s-a)^2-\omega^2}{\{(s-a)^2+\omega^2\}^2}$
$u_a(t)$	$\frac{e^{-as}}{s}$
$\delta_a(t)$	e^{-as}
$(f*g)(t)$	$F(s)G(s)$