

Collaborators

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**Patterns in
molecules and oceans:
linking cellular machinery to
global N:P ratios**

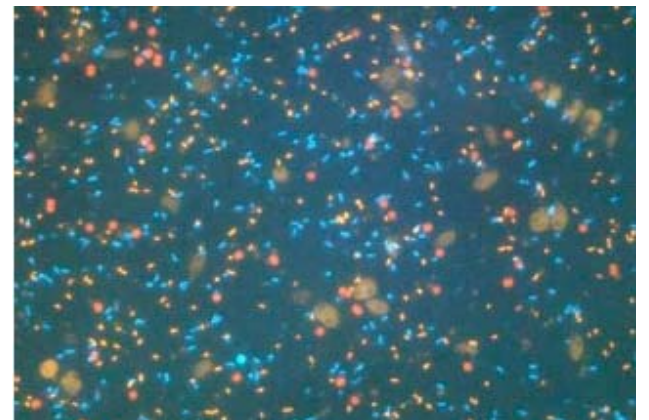
Irakli Loladze

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University of Nebraska - Lincoln
February 4, 2006

- Nitrogen, N
- Phosphorus, P
- N:P

N:P in

- Oceanic Water
and
- Phytoplankton



What is the motivation?

- N and P are essential to all life and globally limit plant and phytoplankton growth
- Phytoplankton plays key roles in climate regulation and biogeochemical cycles



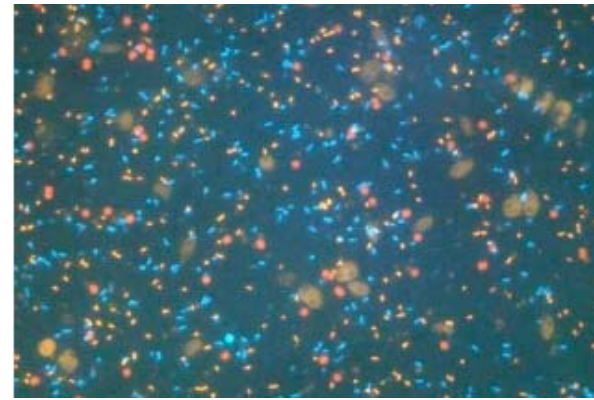
Redfield Ratios

Alfred C. Redfield (1958):
**"The *Biological* Control of Chemical Factors
in the Environment."**



Redfield Ratios

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"The *Biological* Control of Chemical Factors
in the Environment."



Water N:P = Plankton N:P
=16

Is this generally true?

Is Organismal Stoichiometry

=

Environmental Stoichiometry?

IN:

Atmosphere?



Lakes?



Soil?





NO,

NO,

NO

Oceans are kind of

special



Water N:P = Plankton N:P = 16

Water N:P $\text{\textcircled{=}}$ Plankton N:P = 16



A Million dollar Questions:

1) Why " $\text{\textcircled{=}}$ "?

Water N:P = Plankton N:P = 16



A Million dollar Questions:

1) Why "=" in Oceans ?

2) Why 16 ?

Revisiting Redfield...

Water N:P = Plankton N:P = 16?

Revisiting Redfield...

Water N:P = Plankton N:P = 16?

Ocean is big and 3D, so where in ocean
16 is?

Revisiting Redfield...

Water N:P = Plankton N:P = 16?

Ocean is big and 3D, so where in oceans
16 is?

Many species of phytoplankton, so in
what type of phytoplankton 16 is?

Not a pretty picture...

In oceanic water:

Near Bermuda N:P = 30-35, but

near Hawaii N:P = 1-5

In phytoplankton species N:P range is
at least 8-100

Is Redfield true at all, *where is 16?*

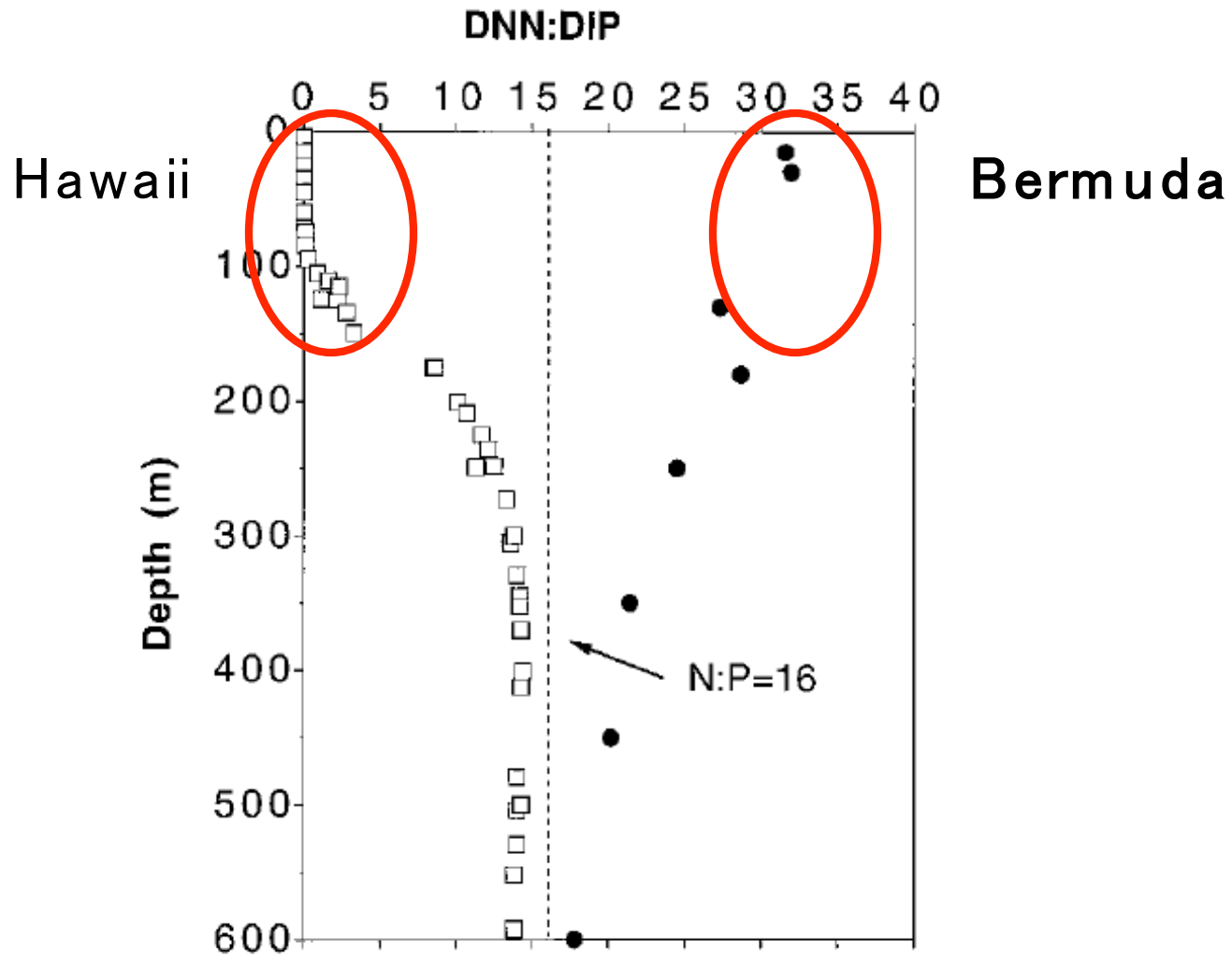


Fig. 2. Vertical profiles of N:P molar ratios in the Sargasso Sea near Bermuda (31.67°N, 64.17°W) (●) and in the Pacific near Hawaii (HOT USJGOFS Web site at hahana.soest.hawaii.edu) (□).

Wu et al (2000) Science, v.289

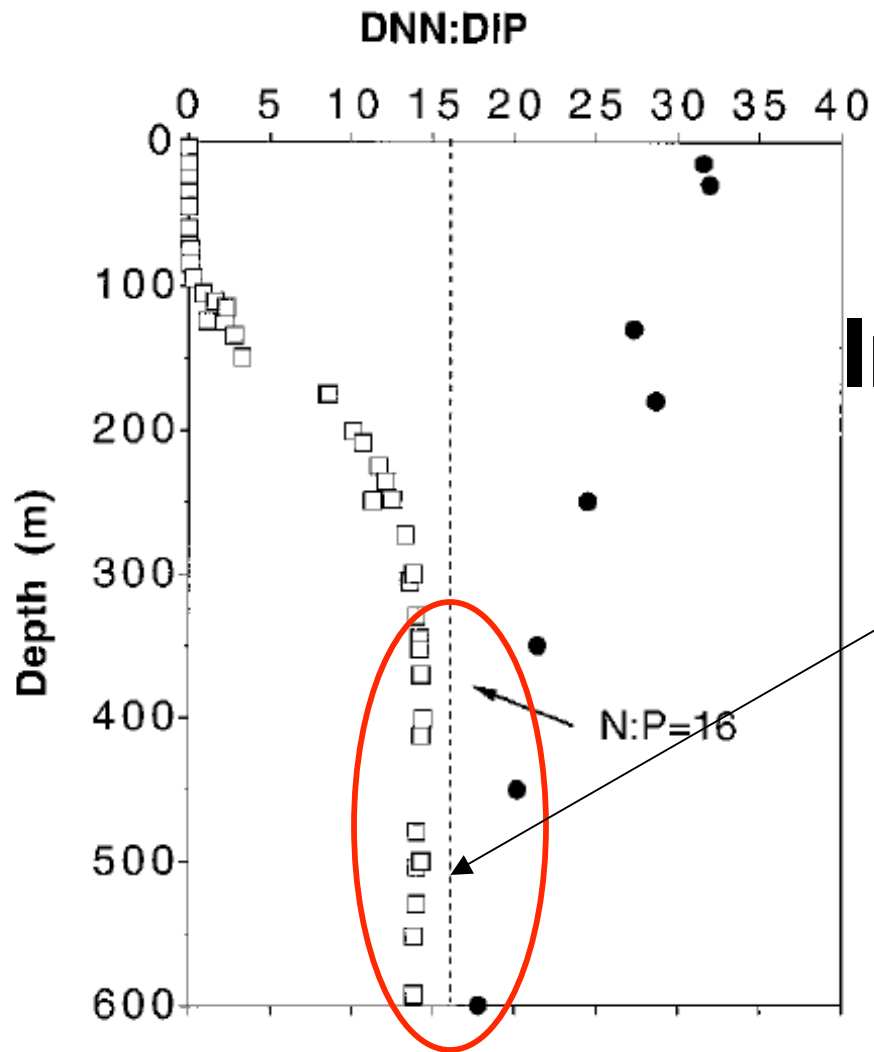


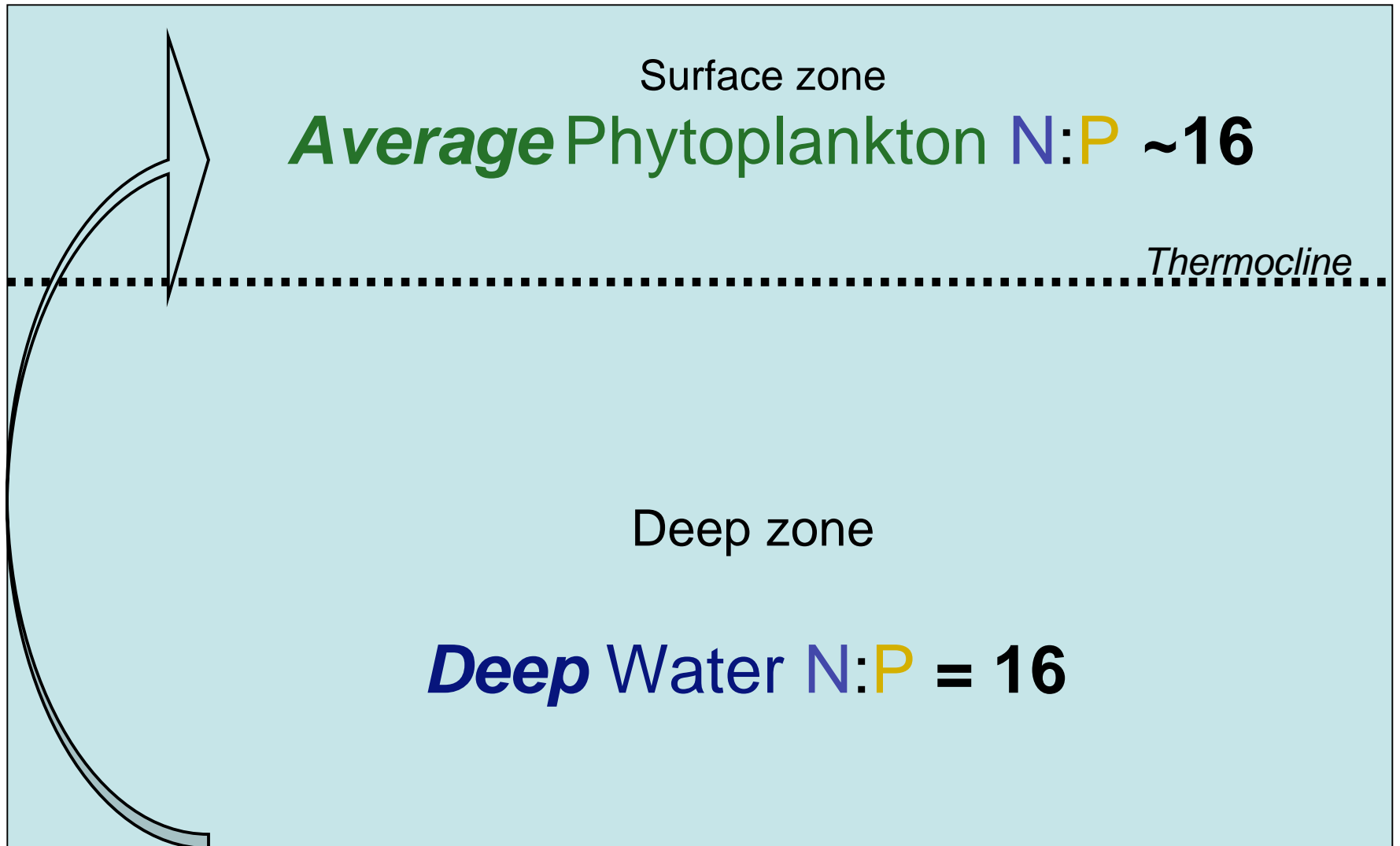
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**Yes, many species of phytoplankton,
but the**

Average Phytoplankton N:P ~16

Ocean 16

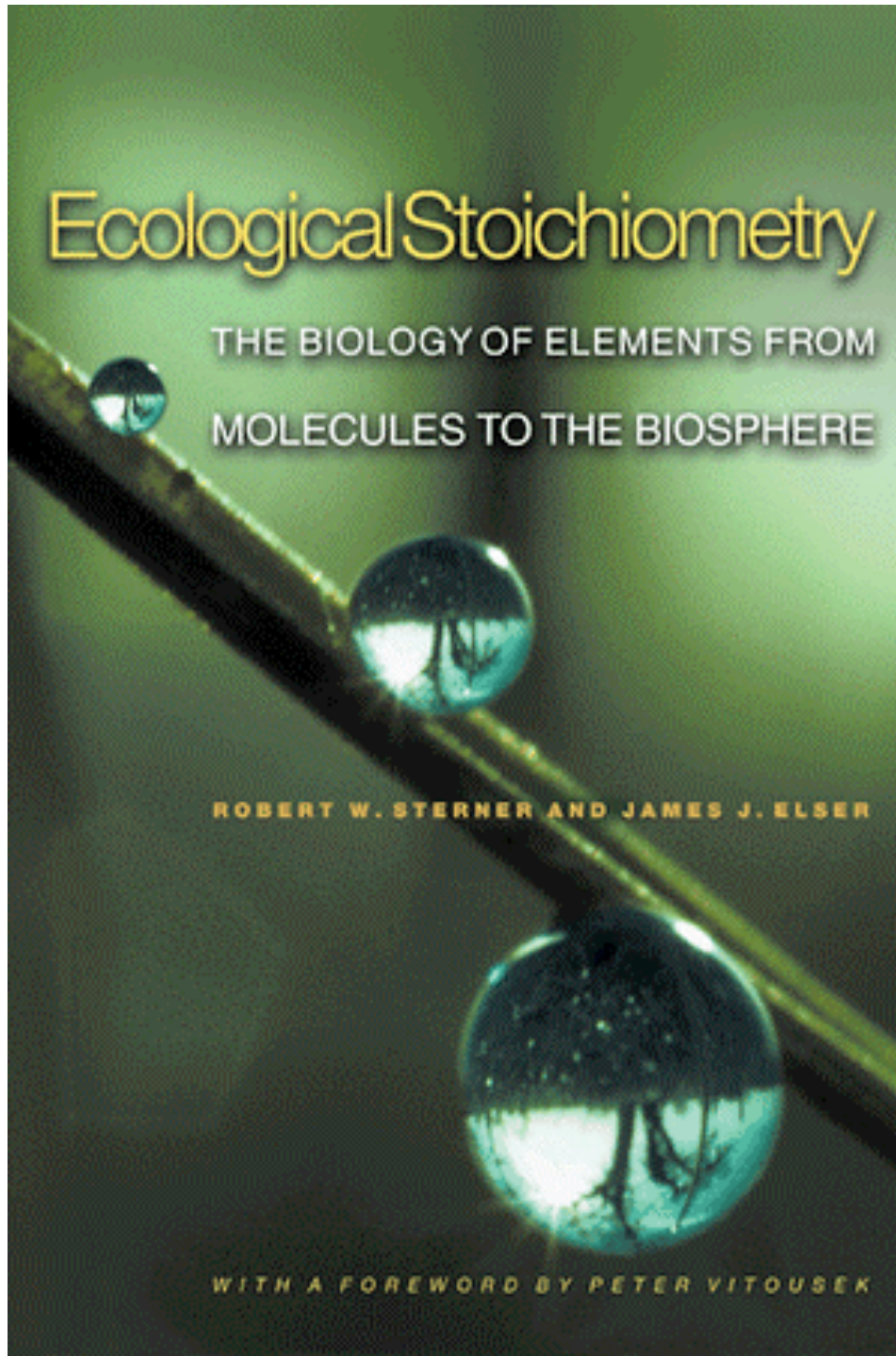


1. Why “=” ?

Average Phytoplankton N:P

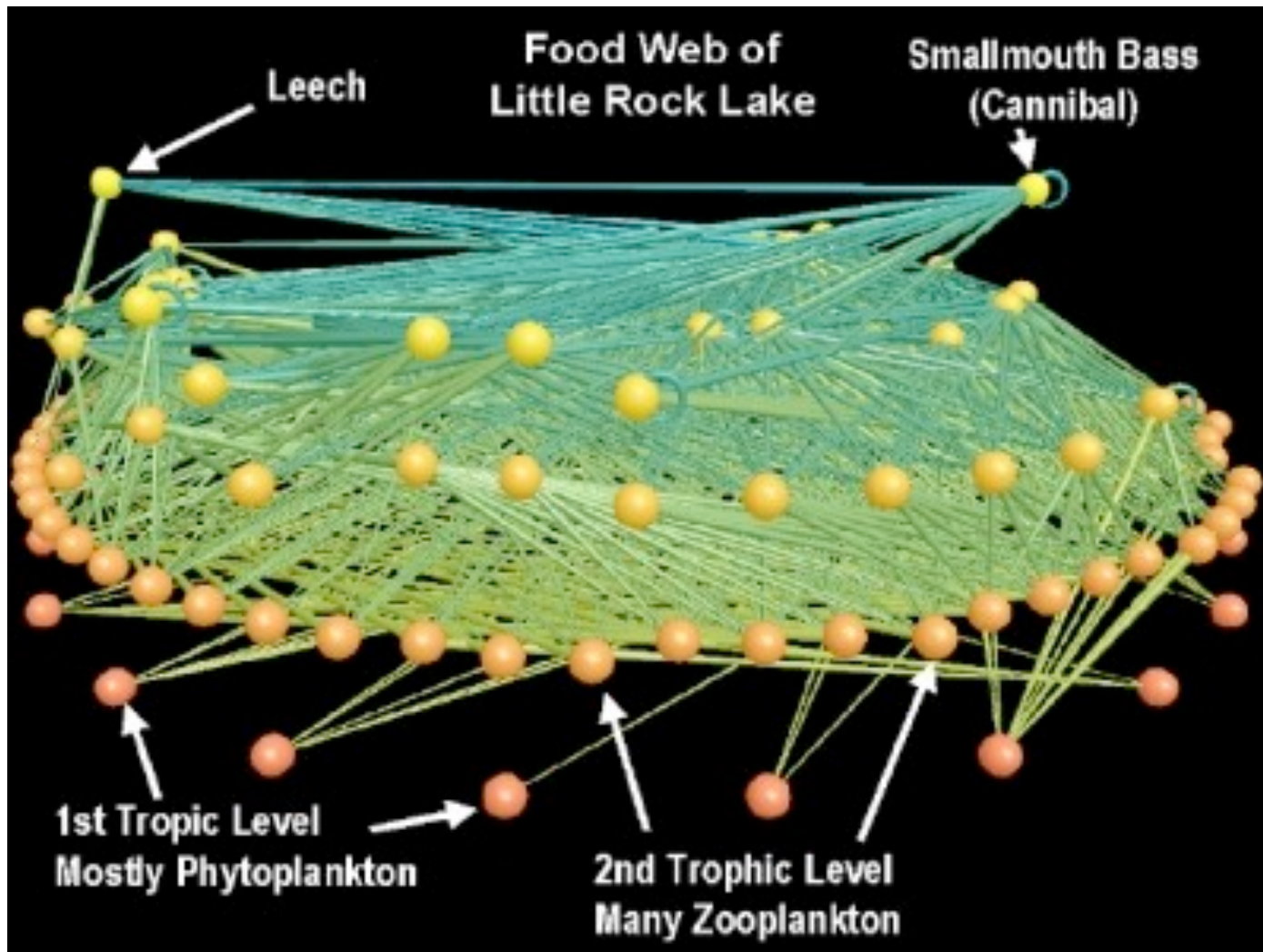
=

Deep Ocean Water N:P ?



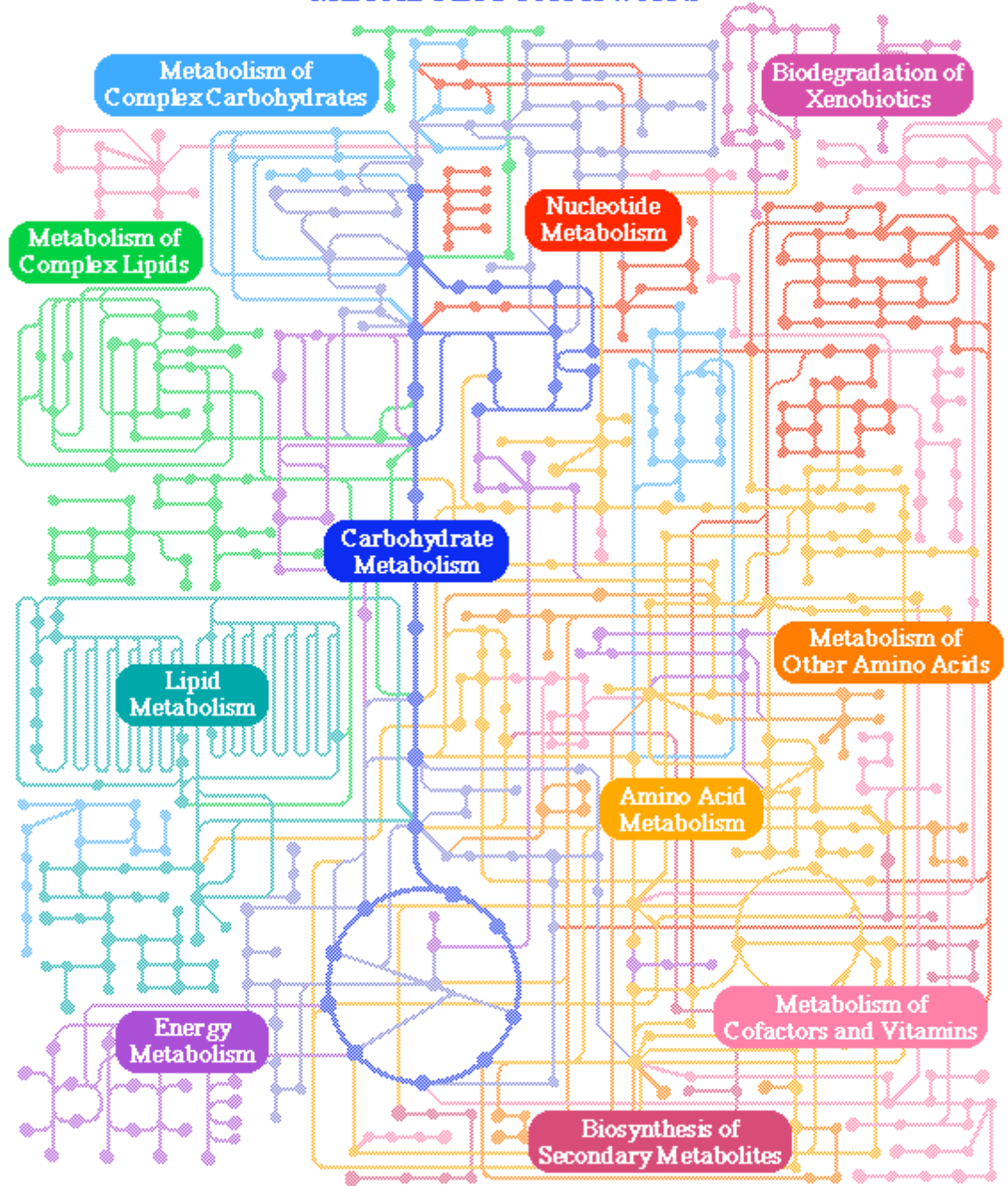
Ecological Stoichiometry

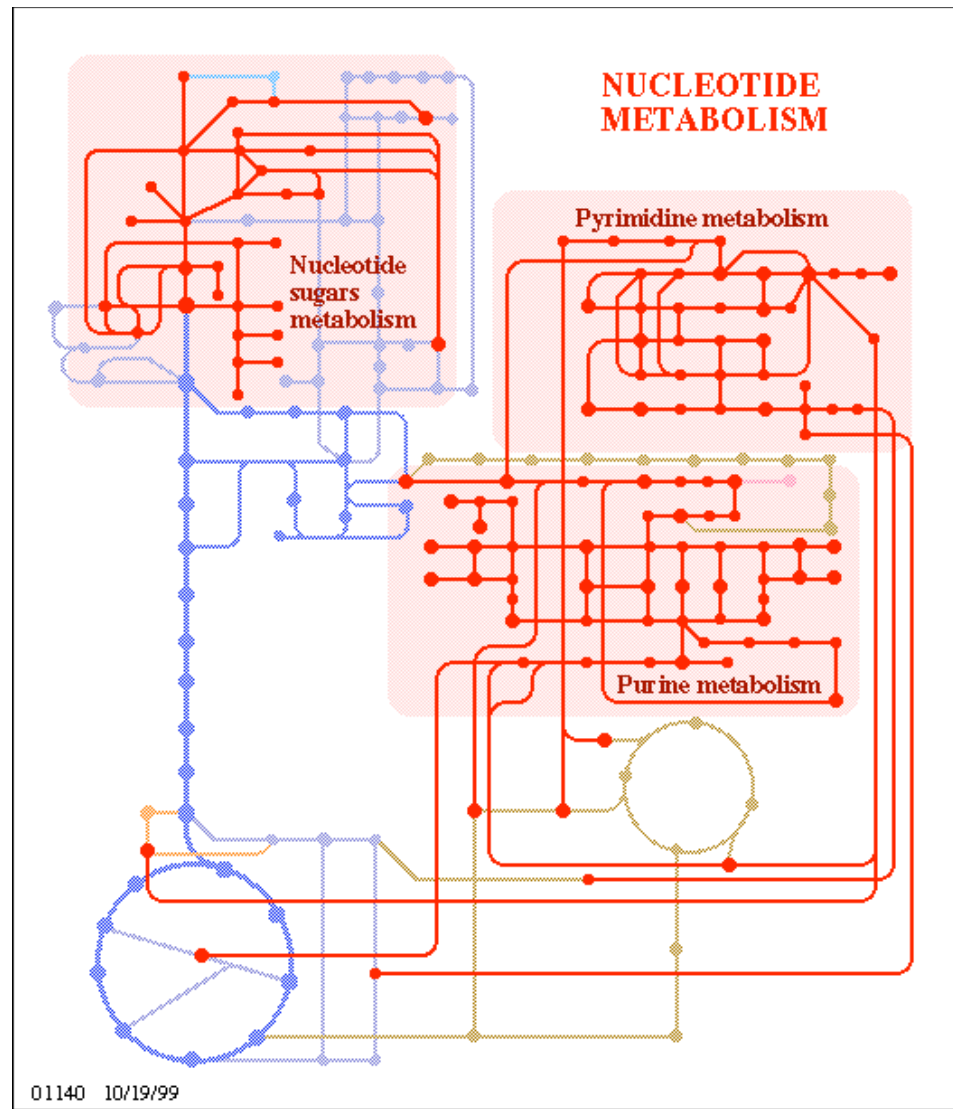
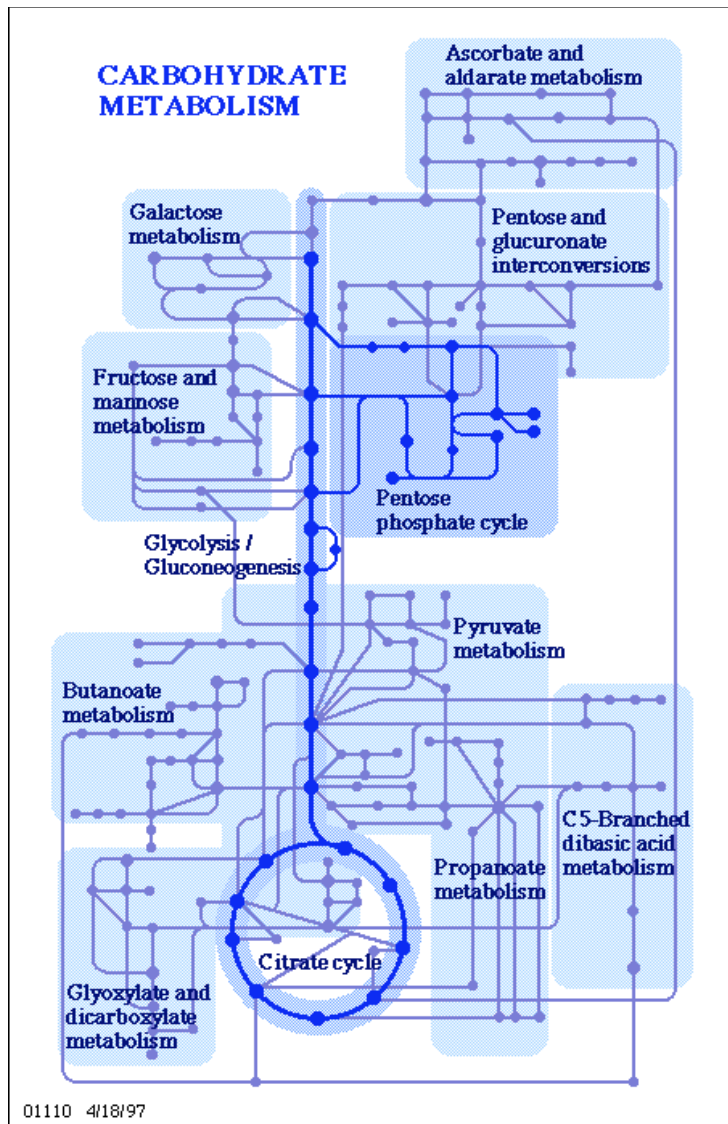
The study of the balance of energy and multiple chemical elements in ecological systems



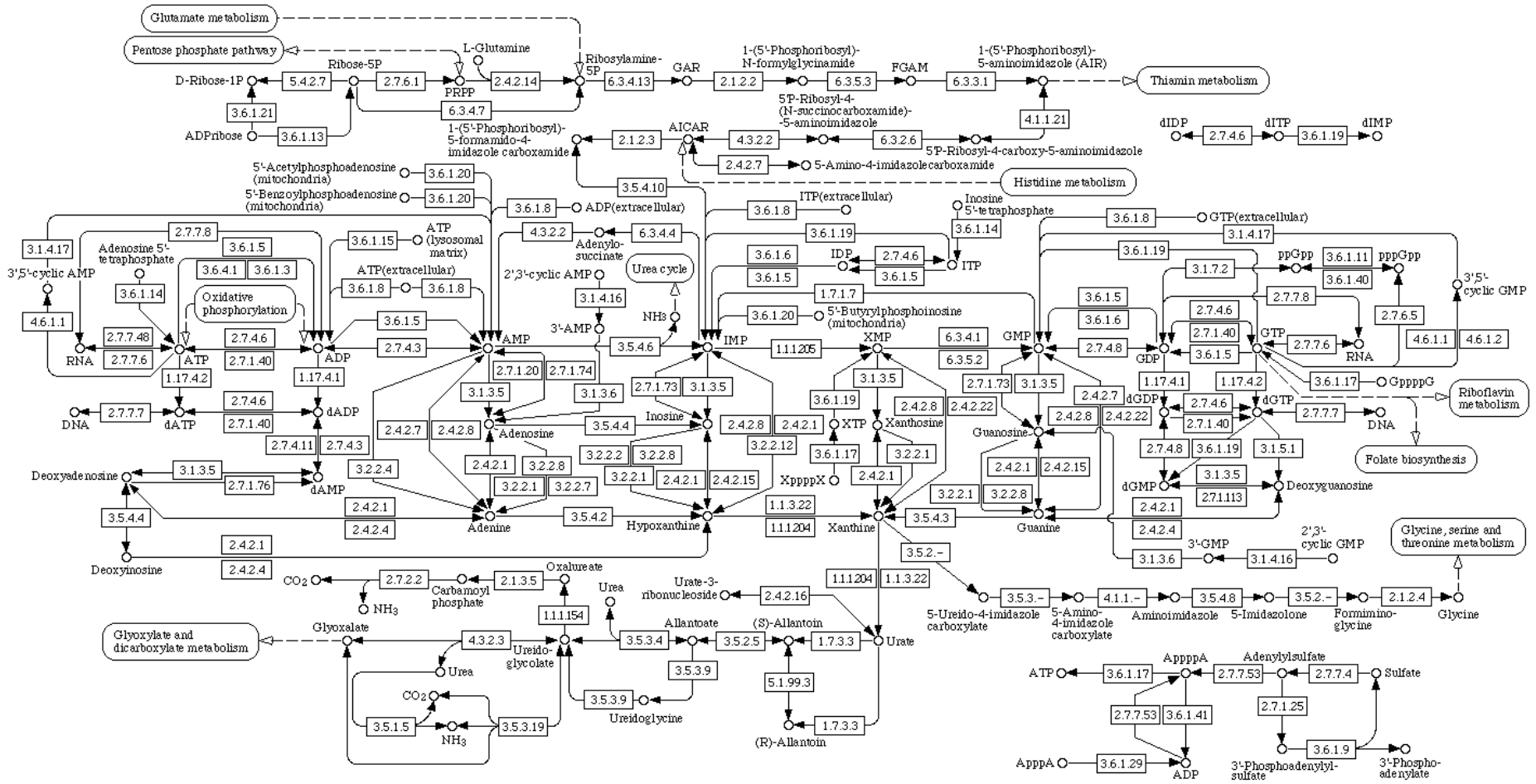
<http://www-personal.umich.edu/~mejn/networks/rock.jpg>

METABOLIC PATHWAYS





PURINE METABOLISM



A couple of Stoichiometric Principles:

- Life **cannot** create, destroy or convert **any** chemical element
- C, H, O, N, and P are essential to **all** Life

**How many biological
systems can be
COMPLETELY
described by these
principles?**

NONE

**For how many
biological systems
these principles are
true?**

AI

1. Why “=” ?

Average Phytoplankton N:P

=

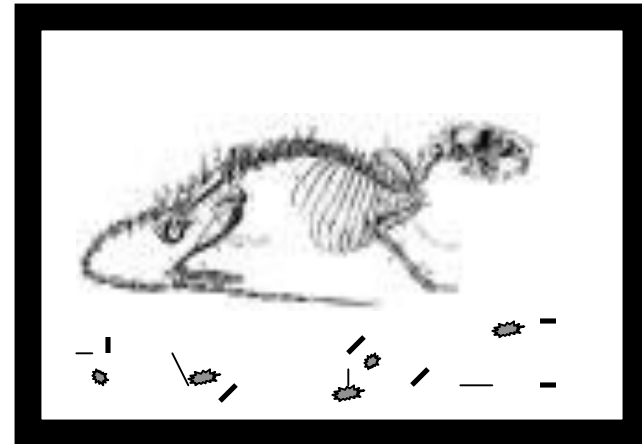
Deep Ocean Water N:P ?

Because of...

Rat in a box



After 500 years...



Stoichiometry = Stoichiometry

Ocean

Average Phytoplankton N:P ~16

Thermocline

Deep Water N:P = 16

"=" because It Sinks!

2) Why **16**?

Average Phytoplankton N:P

=

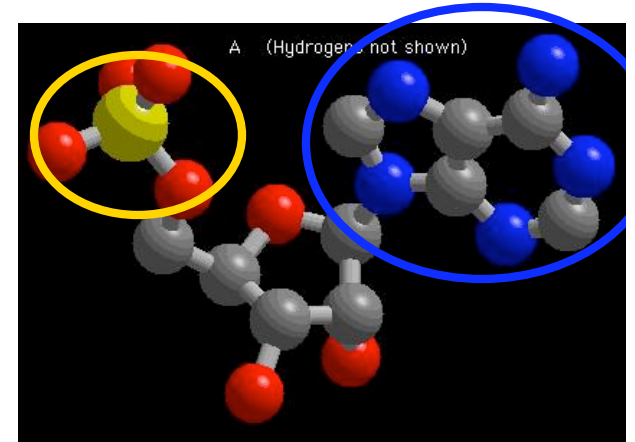
Deep Ocean Water N:P

= **16**

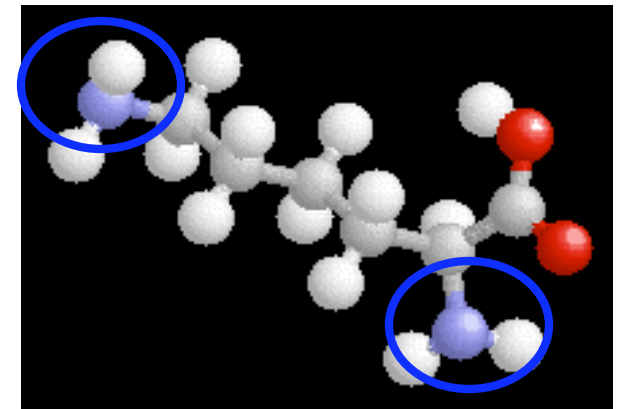
Why not **3, 8, 25, 53, 700?**

Phytoplankton's demand for N and P

- N and P for nucleotides (RNA & DNA)



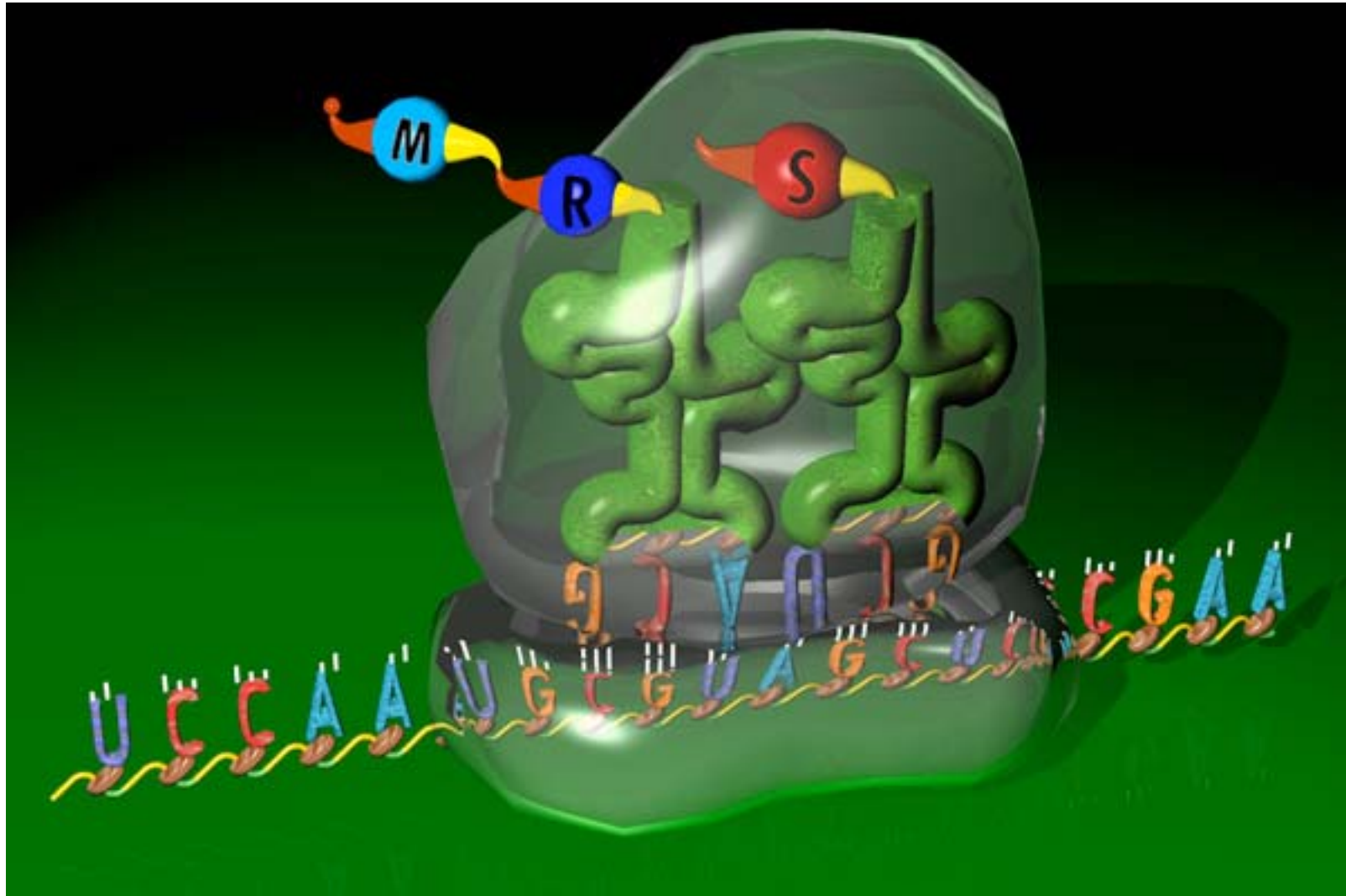
- N for Proteins



FUNDAMENTAL CONSTANTS

Protein Synthesis Machinery

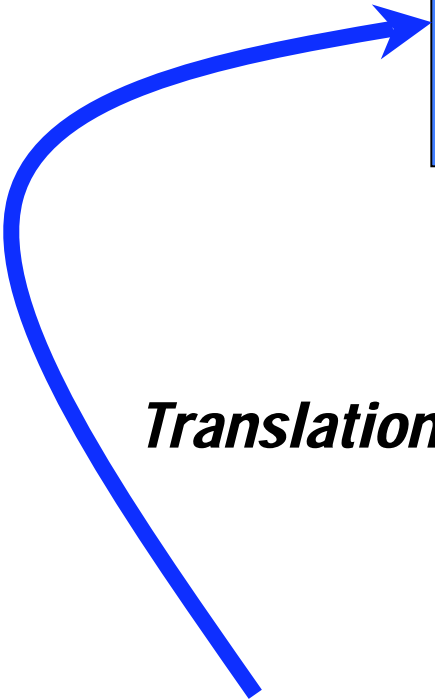
Translation



RNA → Protein

RNA

Protein



Translation λ

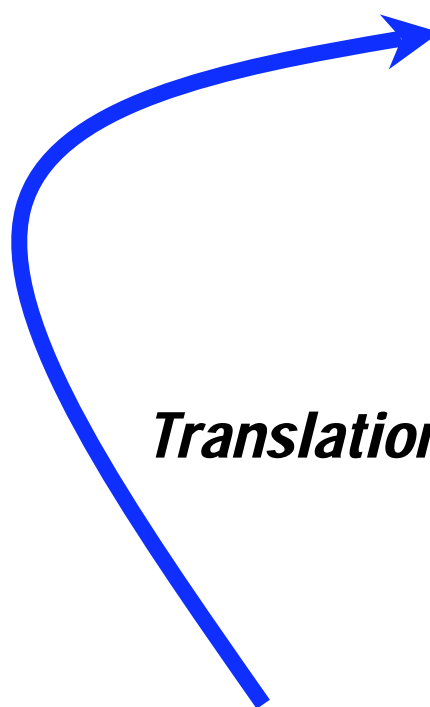
P

N

RNA

$\eta = N\%$ in Proteins, $\sim 17\%$

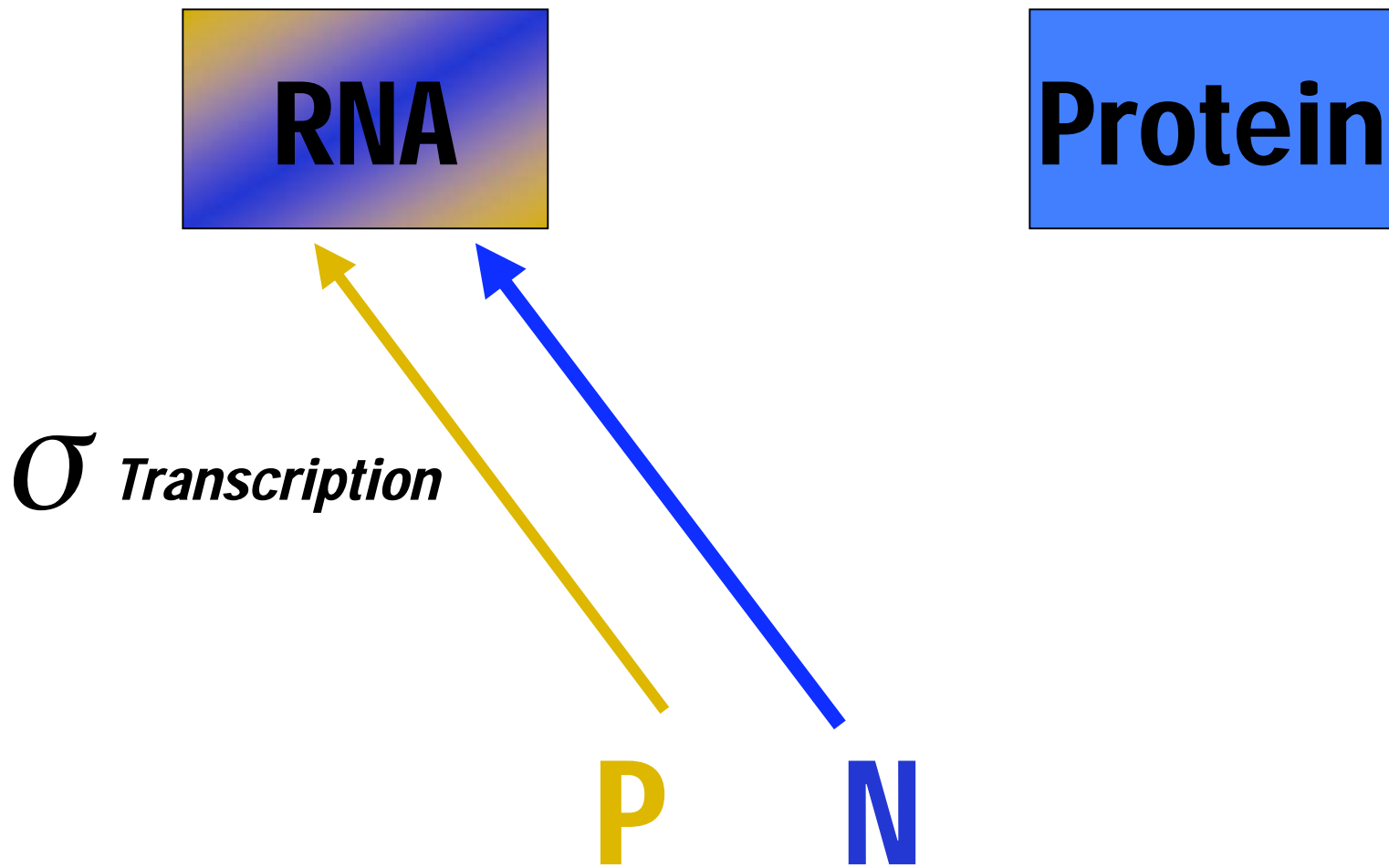
Protein



Translation λ

P

N



Elemental Composition of Nucleotides

- Adenine: C₅H₅N₅
- Guanine: C₅H₅N₅O
- Cytosine: C₄H₅N₃O
- Thymine: C₅H₆N₂O₂
- Uracil (in RNA replaces Thymine) C₄H₄N₂O₂

- Phosphate: PO₄
- 2-deoxyribose: C₅H₉O₄
- (using <http://www.ch.cam.ac.uk/magnus/MolWeight.html>
one can get % of each elements (e.g. for adenine: C₅H₅N₅PO₄C₅H₉O₄)

- Adenine C 33.067; H 3.885; **N 19.281**; O 35.238; **P 8.527**;
- Guanine C 31.672; H 3.721; **N 18.467**; O 37.970; **P 8.167**;
- Cytosine C 31.869; H 4.160; **N 12.388**; O 42.451; **P 9.131**;
- Thymine C 34.006; H 3.995; **N 7.931**; O 45.297; **P 8.769**;
- Uracil C 31.776; H 3.851; **N 8.234**; O 47.031; **P 9.105**;

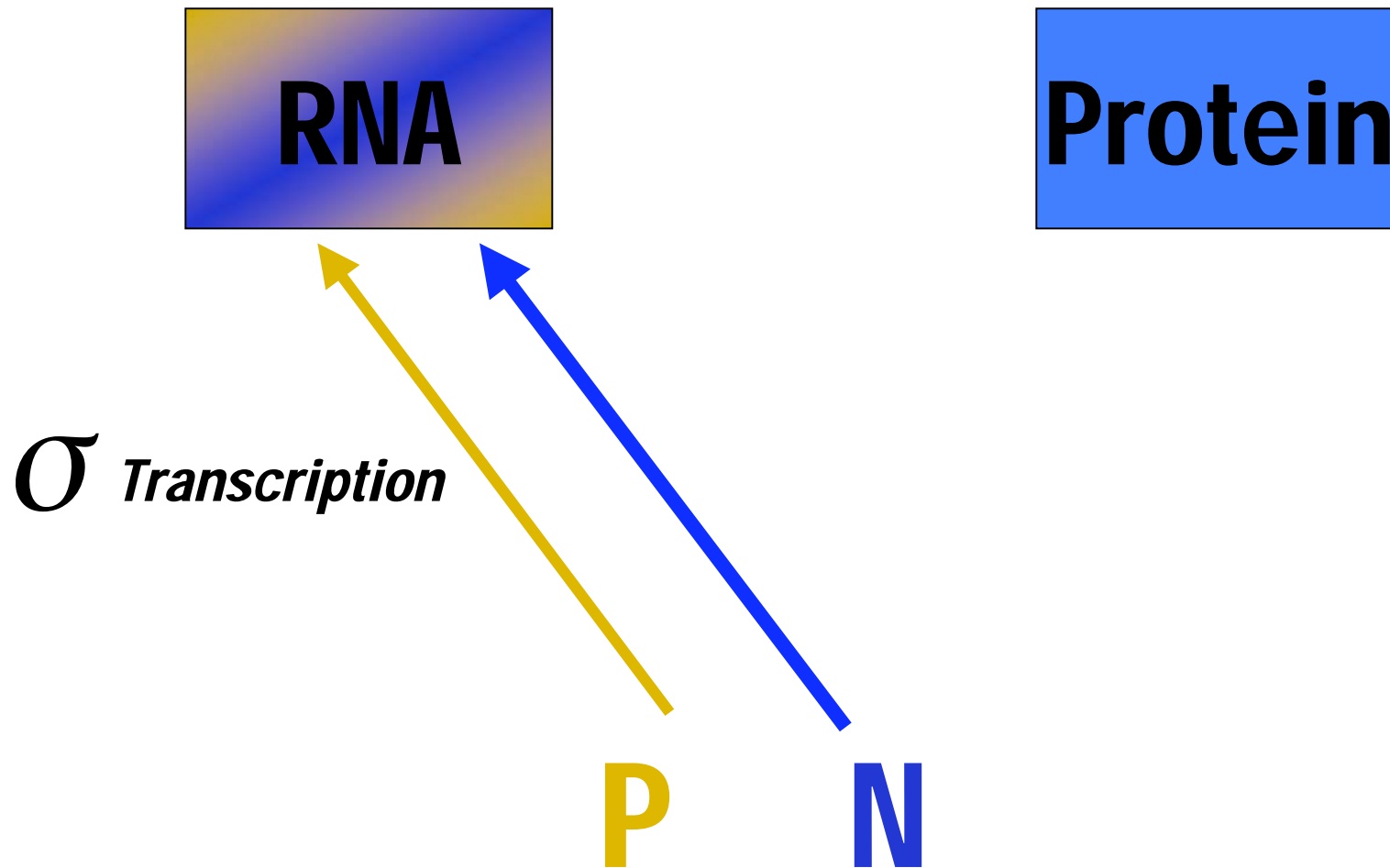
RNA is :

8.7% P,

14.6% N

$\rho = P \% \text{ in RNA, } \sim 8.7\%$

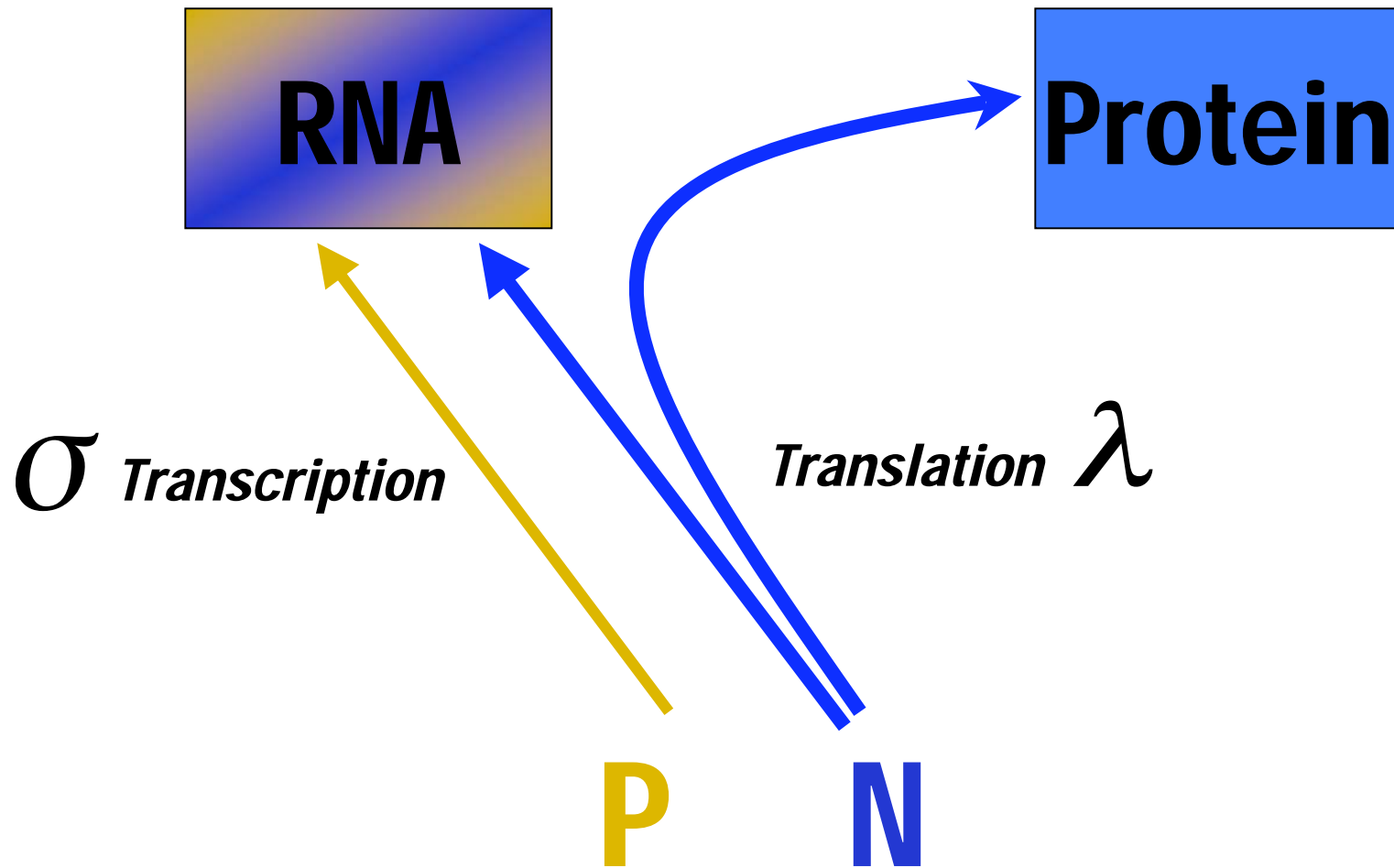
$V = N \% \text{ in RNA, } \sim 14.6\%$



$\rho = P\%$ in RNA, ~14.6%

$V = N\%$ in RNA, ~14.6%

$\eta = N\%$ in Proteins, ~17%



What is biochemically
optimal

Protein:RNA = α ?

Entire Cell = RNA + Protein

Protein:RNA = α

Protein concentration in a cell = $\frac{\alpha}{\alpha + 1}$

RNA concentration in a cell = $\frac{1}{\alpha + 1}$

N:P of a cell = $\frac{\alpha \eta + \gamma}{\rho} = 16?$

Entire Cell = RNA + Protein

Protein:RNA = α

Protein concentration in a cell = $\frac{\alpha}{\alpha + 1}$

RNA concentration in a cell = $\frac{1}{\alpha + 1}$

N:P of a cell = $\frac{\alpha \eta + \gamma}{\rho} = ?$

Parameters:

η is an average N concentration in protein, ~17%

ν is an average N concentration in RNA, ~14.6%,

ρ is an average P concentration in RNA, ~8.7%,

n_{nc} is a number of nucleotides per ribosome,

n_{aa} is a number of amino acids per RNA polymerase,

m_{nc} is an average mass of a nucleotide (310 Daltons),

m_{aa} is an average mass of an amino acid (110 Daltons),

θ is Protein:RNA of ribosomes (~2:3 in eukaryotes and ~1:2 in prokaryotes),

λ_{aa} is physiologically (or chemically) maximal peptide chain elongation rate per one ribosome, ~ 20 aa/sec for bacteria,

σ_{nc} is physiologically (or chemically) maximal RNA chain elongation rate per one RNA Polymerase, ~85 nucl/sec for bacteria,

γ_{pl} is a fraction of active RNA polymerase,

γ_r is a fraction of active RNA

ϕ_{rb} is a fraction of total cell RNA that is ribosomal RNA,

ϕ_{pl} is a fraction of total cell protein that is RNA polymerase.

ϕ_{rp} is a fraction of total cell protein that is r-protein,

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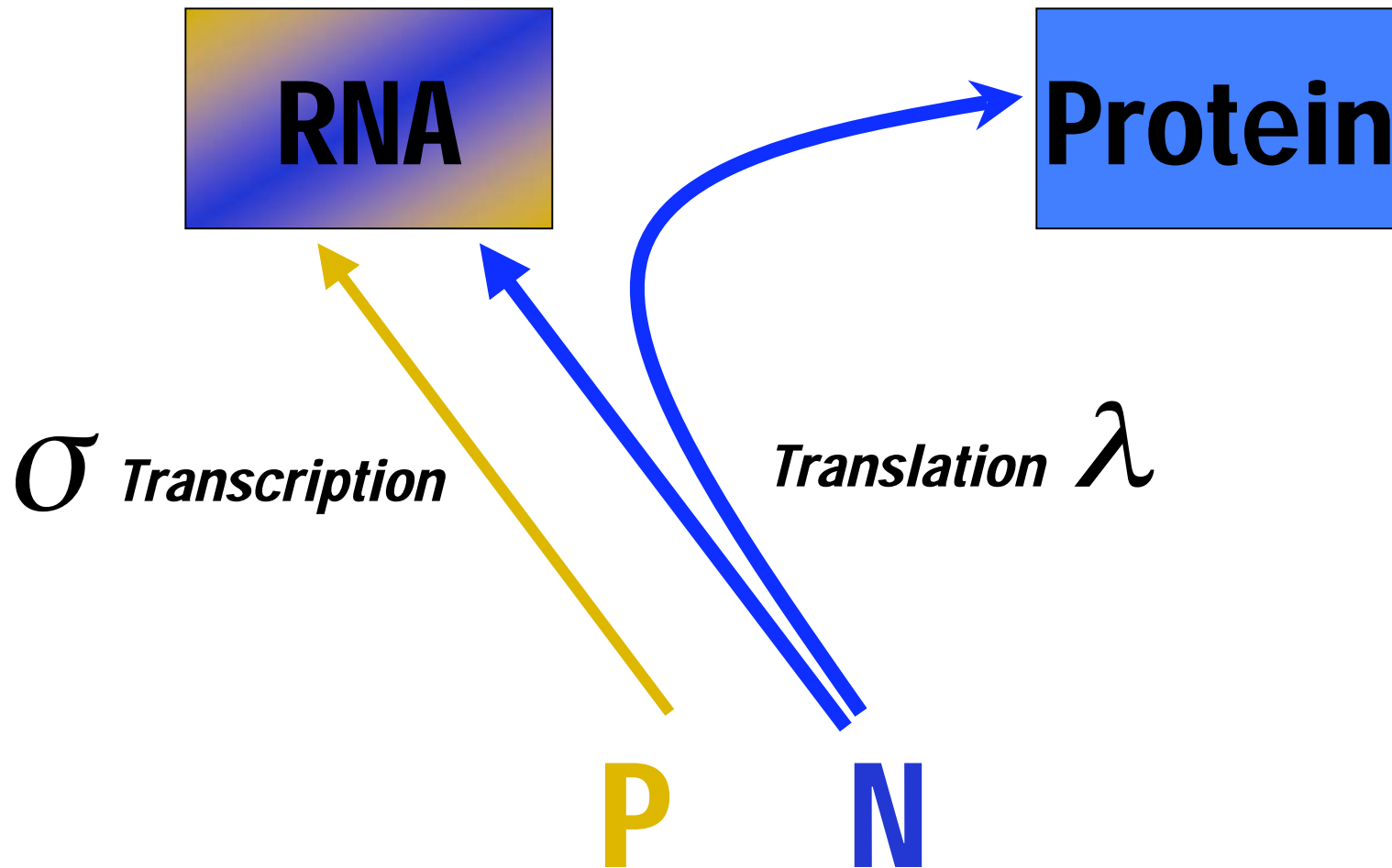
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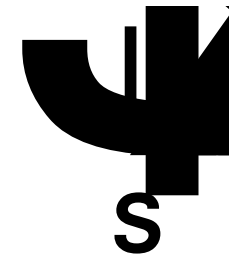
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$V = N\%$ in RNA, ~14.6%

$\eta = N\%$ in Proteins, ~17%



Optimal Protein:RNA = Optimal α =



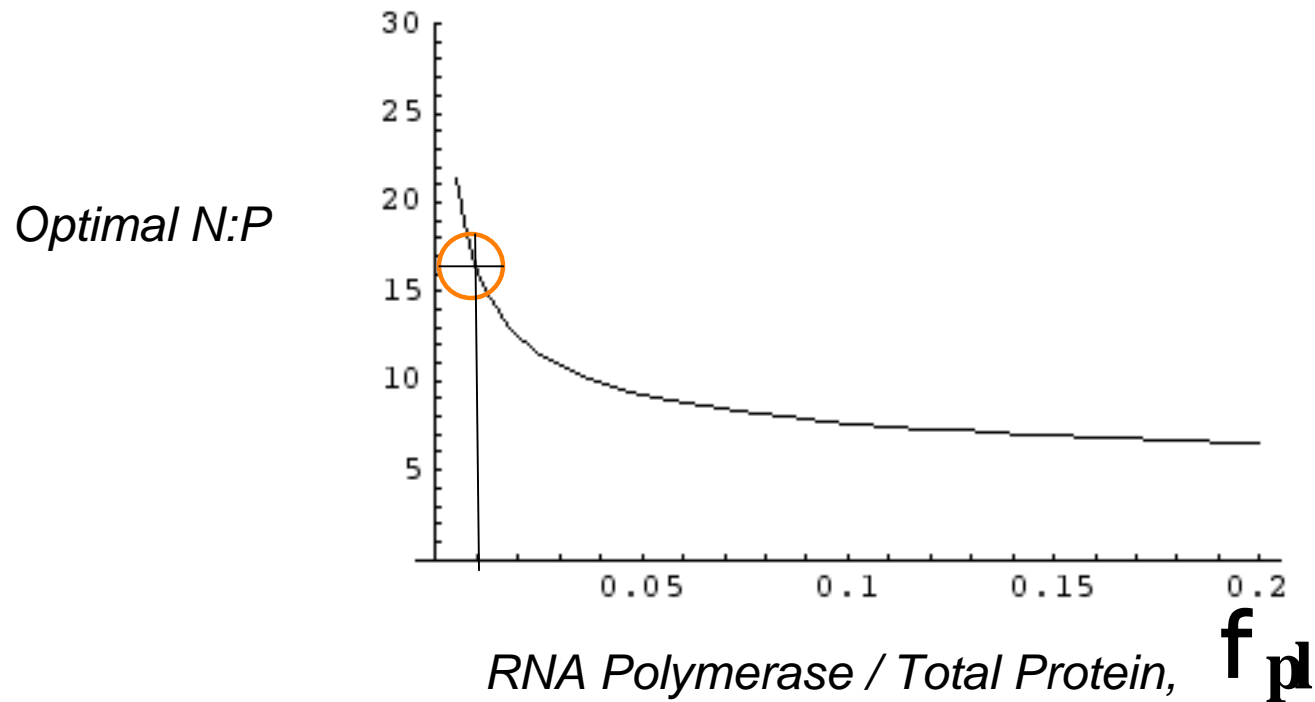
$$\mathbf{N:P} \text{ of a cell} = \frac{\alpha \eta + \gamma}{\rho}$$

**Biochemically optimal N:P ratio
arises from
five fundamental “constants” of life:**

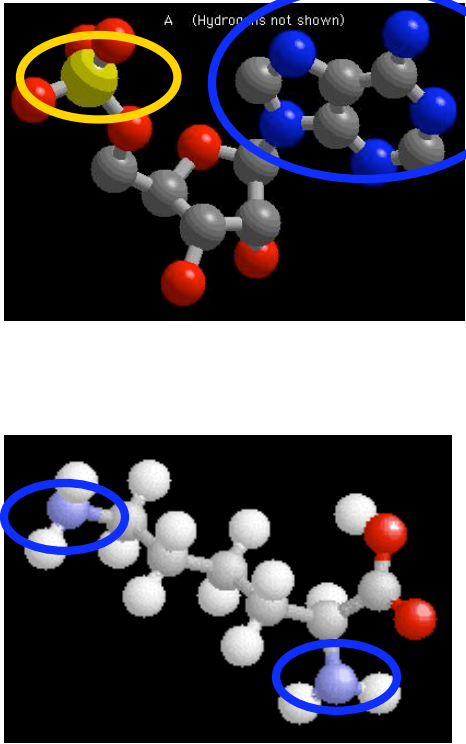
$$N:P = \frac{\sqrt{\lambda/\sigma} \eta + \nu}{\rho}$$

With data from Bremer & Dennis “Modulation of Chemical Composition and Other Parameters of the Cell by Growth Rate”, one gets

$$\mathbf{N:P = 16.13}$$



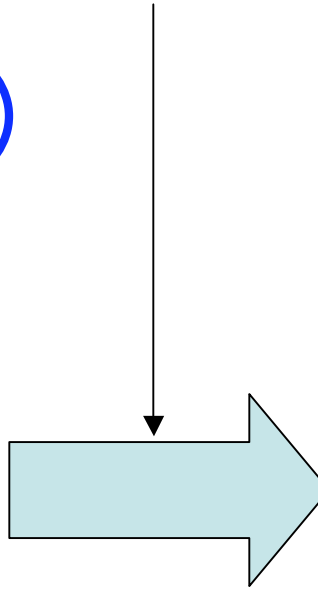
A PROBLEM.



A (Hydrogens not shown)

$$N:P = \frac{\sqrt{\lambda/\sigma} \eta + \nu}{\rho}$$

CELL



Ocean

Adding Evolution

**If species with various
RNA:Protein compete,
who wins?**

Model

$$\frac{dn}{dt} = \delta n_0 - \alpha_n \cdot u(n) \cdot x - \delta n$$

$$\frac{dp}{dt} = \delta p_0 - \alpha_p \cdot u(p) \cdot x - \delta p$$

$$\frac{dn_s}{dt} = \alpha_n \cdot u(n) \cdot x - (1 + \beta^{-1}) \cdot f(n_s, p_s, r) - \delta \cdot n_s$$

$$\frac{dp_s}{dt} = \alpha_p \cdot u(p) \cdot x - \sigma^{-1} \cdot f(n_s, p_s, r) - \delta \cdot p_s$$

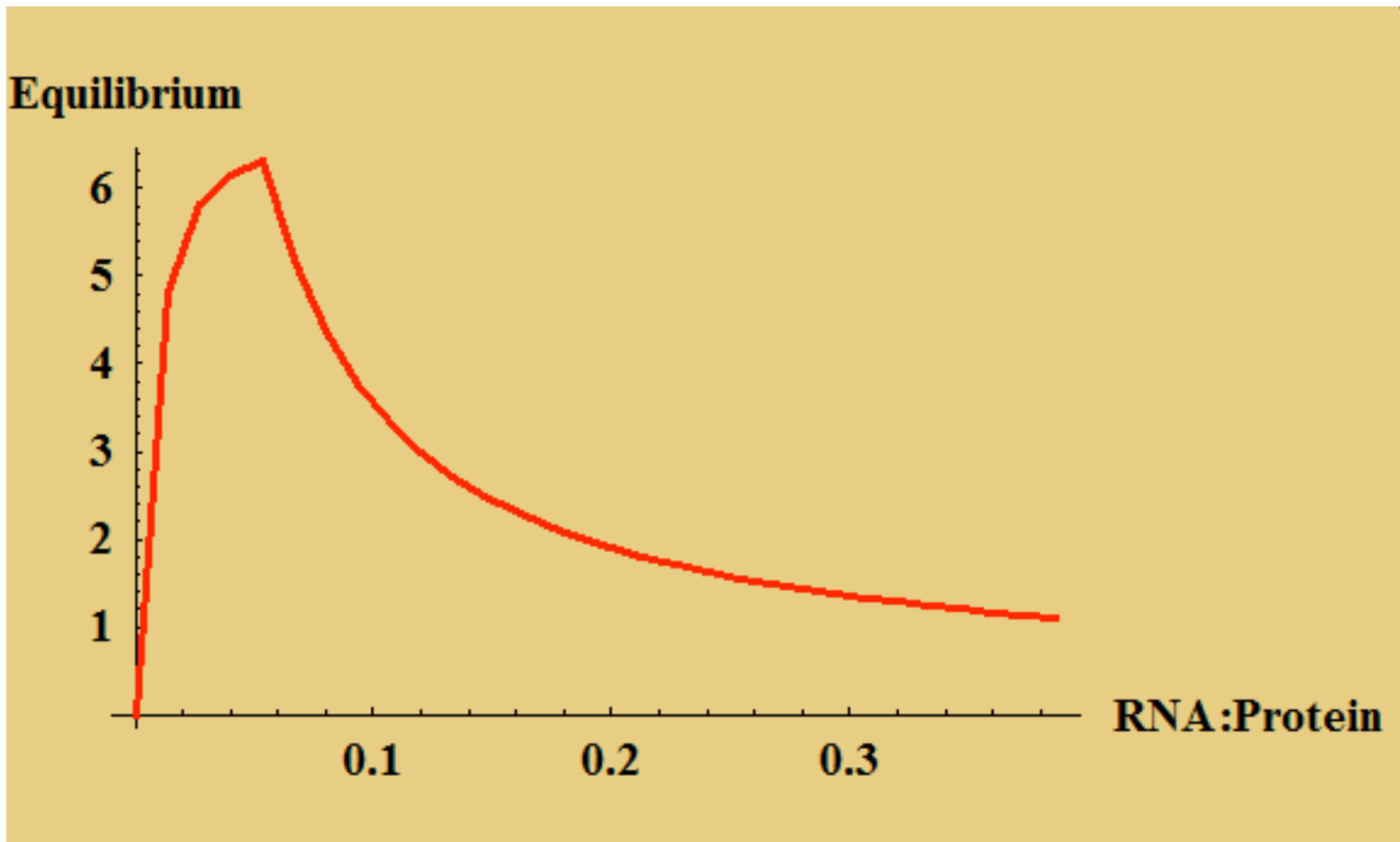
$$\frac{dr}{dt} = \theta_n \cdot f(n_s, p_s, r) - \delta \cdot r$$

$$f(n_s, p_s, r) = \min \{ \beta \cdot \gamma \cdot n_s \cdot r, \psi \min \{ n_s, \sigma p_s \} \cdot x \}$$

$$x = (1 + \beta^{-1})r$$

β is RNA:Protein ratio

Equilibrium dependence on RNA:Protein, but it also depends on incoming N:P



Result

- Evolutionary winner's N:P depends on the incoming N:P

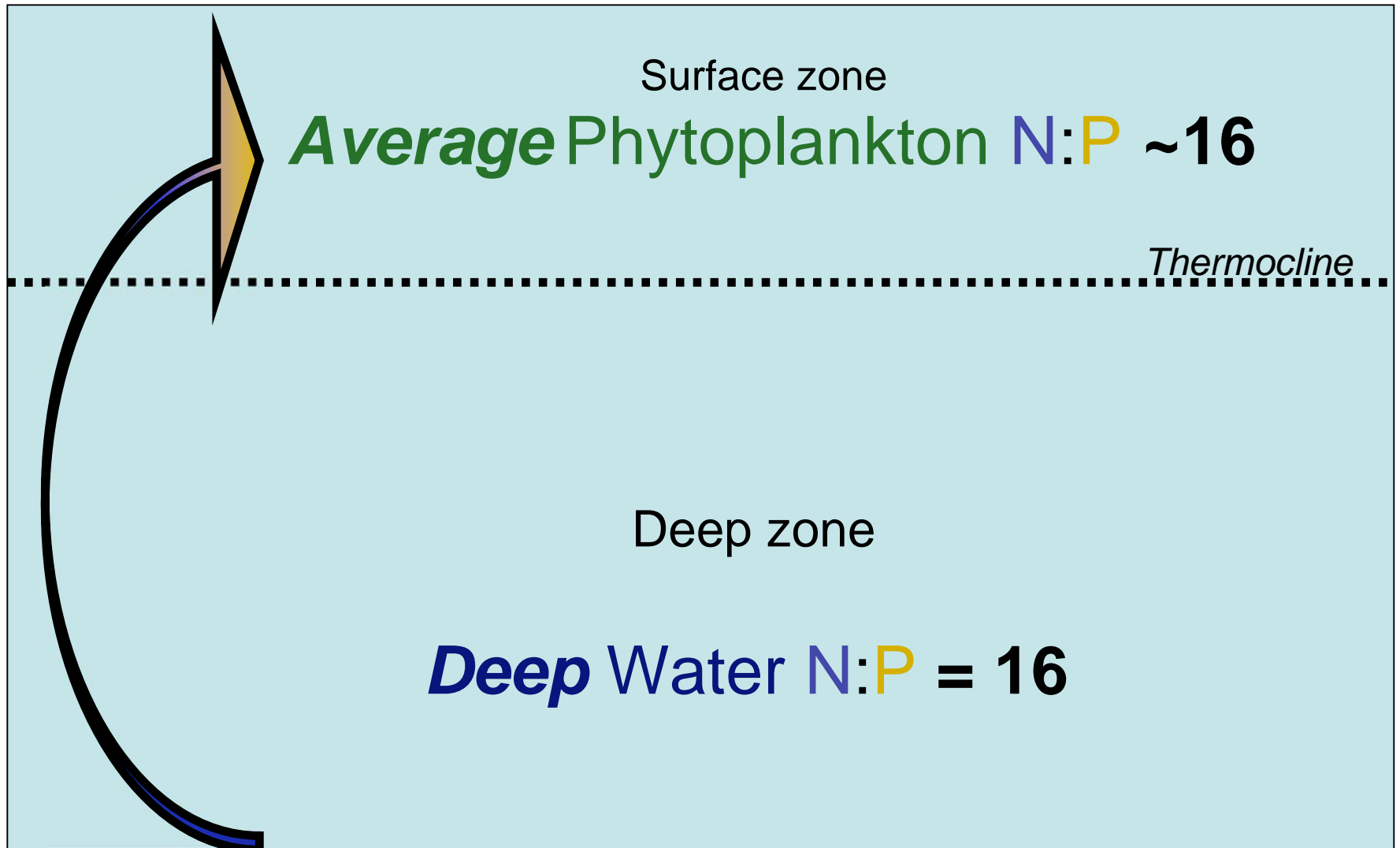
Result

- Evolutionary winner's N:P depends on the incoming N:P

Is this $N:P = \frac{\sqrt{\lambda/\sigma} \eta + \nu}{\rho}$ irrelevant??

Not good. 😞

Ocean 16

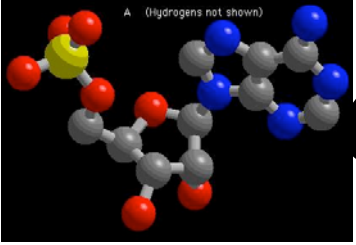


Result!

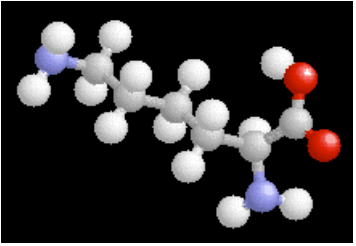
- Once the winner's **N:P** is fed back into the incoming **N:P**, it converges to biochemically optimal **N:P**! 😊

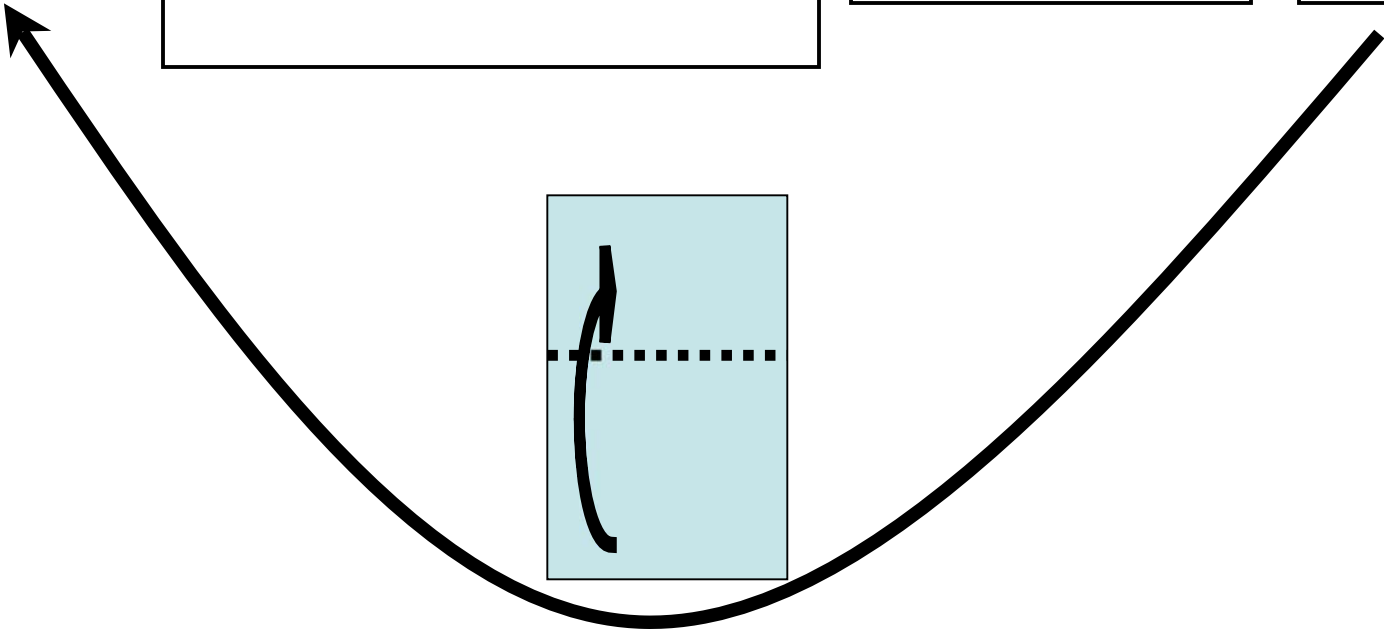
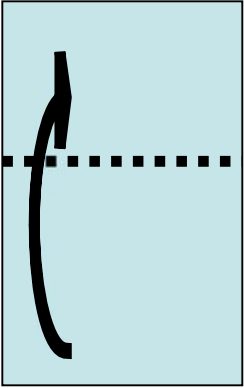
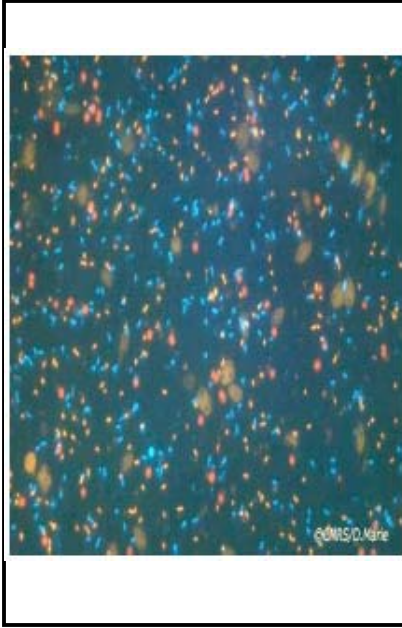
$$N:P = \frac{\sqrt{\lambda/\sigma} \eta + \nu}{\rho}$$

P
N

ν  σ

ρ

η  λ



**THANK YOU for
Listening!**

