

Lecture 2, Thursday, Aug. 23.

Population ecology is a major subfield of [ecology](#)—one that deals with the dynamics of [species populations](#) and how these populations interact with [the environment](#).

http://en.wikipedia.org/wiki/Population_ecology

A single cell of the bacterium *E. coli* would, under ideal circumstances, divide every twenty minutes. ... it can be shown that in a single day, one cell of *E. coli* could produce a super-colony equal in size and weight to the entire planet earth. (**Exercise 1.2**). M. Crichton (1969), *The Andromeda Strain* (Dell, New York, p. 247).

Chapter 2: Single species growth models

2.1. Malthusian and logistic growth models

Required Reading: section 4.1, page 116-121.

Suggested Reading: Thomas Malthus: *An Essay on the Principles of Population Growth*, 1798. <http://www.marxists.org/reference/subject/economics/malthus/>

Suggested Reading: Georgyi Frantsevitch Gause: *The Struggle for Existence*, 1934. Free online <http://www.ggause.com/Contgau.htm>. Buy for \$35 at Amazon.com.

<http://www.amazon.com/gp/product/0486495205/102-6000505-7050558?v=glance&n=283155>

The four major processes that regulate population growths are: birth (B , +), death (D , -), immigration (I , +) and emigration (E , -) (where B is the number of births, D is the number of deaths, I is the number of immigrants and E is number of emigrants). We assume first that the population grows in a closed environment. Hence we will ignore both the immigration and emigration processes.

There are many other factors that keep populations in check such as intra- and inter-specific competition, predation, and diseases. These factors often reduce birth rate and/or increase death rate. Hence we may decompose their effects on population growth into the birth and death processes. We assume that the population change occur **continuously**.

$$dN/dt=B-D. \quad (\text{Eqn 4.1})$$

The alternative is to use **difference** equations -- with that technique, time changes **discretely**. An example of a difference equation for population growth is $N_t = 2 * N_{t-1}$.

If we assume that birth rate and death rate are constant b and d , respectively, in equation 4.1, then we obtain ($r=b-d$)

$$dN/dt=bN-dN=(b-d)N=rN. \quad (\text{Eqn 4.2})$$

Equation describes an exponentially growing population. This equation is also referred as *Malthusian growth model*. $N(t)$ in terms of our starting population, $N(0)=N_0$, and the growth rate r takes the form of

$$N(t) = N_0 e^{rt}. \quad (\text{Eqn 4.3})$$

A quantity that is sometimes of interest is the **doubling time** — the time it takes a population to double in size under positive exponential growth.

$$2N_0 = N_0 e^{rt}. \quad (\text{Eqn 4.4})$$

We can cancel the N_0 , then take the log of both sides, giving us $\ln(2) = rt$ or

$$t = \ln(2)/r. \quad (\text{Eqn 4.5})$$

r is sometimes called the **intrinsic or instantaneous rate of increase**. It expresses the balance between birth and death processes. Here are some conditions under which populations **may** grow exponentially for a short period of time.

- 1) **Invasive** species when they first arrive.
- 2) Species **colonizing** a new habitat (e.g., an isolated island).
- 3) Species that are **rebounding** from a population crash.
- 4) When they develop **novel adaptations** to cope with the environment (cancer cells).

It can be shown that in a single day, one cell of E. coli could produce a super-colony equal in size and weight to the entire planet earth. Why this never took place? The hidden and false assumption here is that the birth and death rates are constant -- that is, we implicitly assumed that the birth and death rates were independent of the population size and wouldn't change over time. The simplest (ad hoc) way to correct these assumptions is to assume these rates are linearly dependent on population density:

$$b = b(N) = b_0 - b_1 N, \quad d = d(N) = d_0 + d_1 N. \quad (\text{Eqn 4.6})$$

This yields the following logistic growth model (where $K = r / (b_1 + d_1)$, $r = b_0 - d_0$):

$$dN/dt = b_0 - b_1 N - d_0 - d_1 N = rN(1 - N/K). \quad (\text{Eqn 4.7})$$

The solution of the logistic equation takes the form of:

$$N_t = \frac{N_0 e^{rt}}{1 + (e^{rt} - 1) \frac{N_0}{K}}. \quad (\text{Eqn 4.8})$$

Below is a typical solution of the logistic population growth ($K=100$). We observe that at population size of $K/2$, the growth rate begins to decline and eventually reaches an asymptote at

the carrying capacity, K . Changing the value of r will affect the steepness of the ascending portion.

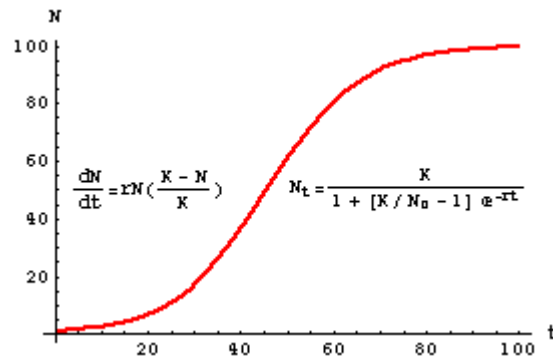


Figure 2.1. A typical solution of the logistic population growth ($K=100$).

Questions to ponder: 1) What happens when $N > K$? 2) At what point is the population growing most rapidly? How would you find that mathematically?

Exercise 1.3: Problem 5, page 152.

Exercise 1.4: If this were a harvested population, where would you like to maintain the population size in order to manage for Maximum Sustained Yield (MSY)? (Hint: Assume that the population is harvested at the rate of C /unit of time, then the population grows according to $dN/dt = rN(1-N/K) - C$.)

Many observed population growth dynamics can not be appropriately modeled by logistic model (Eqn 4.7). **Other models are needed.**

How we obtain r and K from real-world data on observed population sizes over time? With an observed set of measurements of population size against time, we can estimate r by plotting the data with X -axis, t and Y -axis, population sizes at various times t , and then: 1) eyeballing an estimate of K (the asymptote or place where the curve flattens out); 2) since we now have an estimate of K (and already knew N_0 and N_t), we can solve the log transformed logistic equation:

$$\ln \frac{K - N_t}{N_t} = -rt + \ln \frac{K - N_0}{N_0} \quad (\text{Eqn 4.9})$$

for r ; 3) or, we can estimate r graphically, as (the negative of) the slope of the plot of Eqn 4.9.

For example, Gause fit his experiment data on the growth of *Paramecium caudatum* to logistic model and yielded the saturating population level at $K = 375$ individuals. The coefficient of multiplication or the biotic potential of one *Paramecium* (r) was found to be 2.309. This means that per unit of time (one day) under his experiment conditions of cultivation, every *Paramecium* can potentially give 2.309 new *Paramecia*.

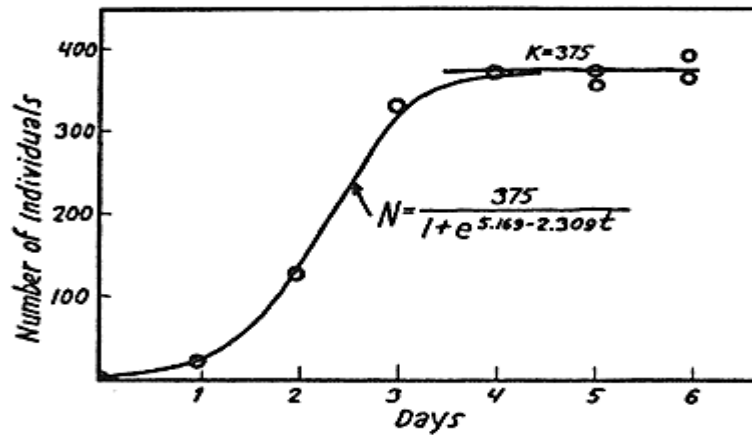


Fig. 2.2. The growth of population of *Paramecium caudatum* (Fig. 4 in Gause).

Regrowth equation $N' = rN - rN^2/K$ is a preferred choice when modeling tree and perennial plant growth. Recently, it is often used to model immune cell growth and late stage resource (such as space and some limiting nutrient) limited growth.