Homework 5 for MAT 494 Due on Th. Nov. 30, 2006

(1) (This combines exercises 3-6 in reference 1, page 287-288) An influenza epidemic was reported at an English boarding school in 1970 which spread to 512 of the 763 students.

1. Estimate the basic reproductive number for this influenza epidemic.
2. What fraction of the boarding school students would have had to be immunized to prevent such an epidemic?
3. What was the maximum number of the boarding school students suffering from influenza at any time?

(2) Consider the model

$$\begin{aligned}\frac{dS}{dt} &= \mu N - \beta S\frac{I}{N} - \mu S + \delta I\\ \frac{dE}{dt} &= \beta S\frac{I}{N} - (\mu+\gamma)E\\ \frac{dI}{dt} &= \gamma E - (\mu+\delta)I\end{aligned}$$

Explain the model and its assumptions as well as the meaning of each parameter. Find R_0, give its biological interpretation and study the stability of the disease free state.

(3) Consider a population experiencing two competing infections, that is, different strains of the same bacterial infection,

$$\begin{aligned}\frac{dS}{dt} &= \mu N - \beta_1 S\frac{I_1}{N} - \beta_2 S\frac{I_2}{N} - \mu S + \gamma_1 I_1 + \gamma_2 I_2,\\ \frac{dI_1}{dt} &= \beta_1 S\frac{I_1}{N} - (\mu+\gamma_1)I_1,\\ \frac{dI_2}{dt} &= \beta_2 S\frac{I_2}{N} - (\mu+\gamma_2)I_2.\end{aligned}$$

1. Explain the biological assumptions of this model including the definitions of each parameter.
2. Can both infections co-exist? Explain your answer.

(4): Exercise 6, page 363.

(5): Exercise 19, page 368.

(6): Exercise 21, page 368.