Homework 3 for MAT 494 Due on Th. Oct. 19, 2006
For problems 1-4, all parameters are assumed to be positive

1): Consider the Verhulst difference equation

$$x_{n+1} = \frac{rx_n}{x_n + a},$$

with $x_0 > 0$. Show that $\lim_{n\to\infty} x_n = 0$ if $r \leq a$, and $\lim_{n\to\infty} x_n = r - a$ if $r > a$.

2): Consider the the following nonlinear difference equation

$$x_{n+1} = \frac{rx_n^2}{x_n^2 + a^2},$$

with $x_0 > 0$. Show that $\lim_{n\to\infty} x_n = 0$ if $r < 2a$, and $\lim_{n\to\infty} x_n = (r + \sqrt{r^2 - 4a^2})/2$ if $r > 2a$ and $x_0 > (r - \sqrt{r^2 - 4a^2})/2$.

3): We can consider a population that consumes its own juveniles in order to self-regulate its size. Assume that the encounter of juvenile and adult individuals follow a Poisson distribution, then the probability that a juvenile will escape from being cannibalized is e^{-cx_2} when the adult prey population is of size x_2 for some $c > o$, where c may be interpreted as the cannibalism rate. The birth rate of the adult population is assumed to be a constant $b > 0$. The survival probability of the juvenile population is density dependent and depends only on its own population size. For simplicity, a Holling type II function can be used to model this nonlinearity. Then a plausible model may take the form of

$$\left\{ \begin{array}{rcl} x_1(t+1) & = & bx_2(t)e^{-cx_2(t)}, \\ x_2(t+1) & = & \dfrac{ax_1(t)}{1 + mx_1(t)}. \end{array} \right. \tag{1}$$

a) Reduce (1) into a equation of the form $x_2(t+2) = f(x_2(t))$ and then study the existence and local stability of the nonnegative steady states of $y(t+1) = f(y(t))$ where $y_t = x_{2t}$.
b) It is easy to show that the function f in a) has a single peak (where $f'(x) = 0$ and $f''(x) < 0$). Try to obtain some global stability results based on the peak locations by applying the theorems and method of lecture note 8-9 (http://math.la.asu.edu/%7Ekuang/class/494/Lecture8.pdf).

4): We briefly covered the paper "Is there a sigmoid growth of Gause's paramecium caudatum in constant environment", by Zufei Ma, Dianmo Li and Baoyu Xie on Thursday, Sept. 28. It can be downloaded at
http://aimsciences.org/journals/pdfs.jsp?paperID=541&mode=full
Try to fit the full loop data with the Ricker equation (Ricker (1954)):

$$N_{n+1} = N_n \exp r(1 - N_n/K), \qquad x_0 > 0. \tag{2}$$

How you compare your fitting with that of Ma et al.?

5): Exercise 12, page 154.
6): Exercise 14, page 154.
7): Exercise 25, page 157.
8): Exercise 26, page 158.
9): Exercise 29, page 160.
10): Exercise 5 and 6 ((a), (f), (d)), page 201.
11): Exercise 7 ((a), (b), (e)), page 201.