## Lecture 2, Tuesday, Aug. 29.

# Chapter 2: Single species growth models, continued

## 2.2. Linear difference equations, Fibonacci number and golden ratio.

***Required Reading:*** *The whole chapter 1.*

***Suggested Reading:*** *http://en.wikipedia.org/wiki/Fibonacci_number*

We shall study the solution structures of general linear difference equations. One of the most well known examples of such equations is Fibonacci equation F(n+1)=F(n)+F(n-1).

Leonardo Fibonacci was born in Pisa, Italy, around 1175. He was the first to introduce the Hindu - Arabic number system into Europe. Leonardo wrote a book on how to do arithmetic in the decimal system, called "Liber abaci", completed in 1202. It describes the rules we are all now learn at elementary school for adding numbers, subtracting, multiplying and dividing. A problem in the third section of Liber abaci led to the introduction of the Fibonacci numbers:

<u>A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?</u>

By charting the populations of rabbits, Fibonacci discovered a number series from which one can derive the Golden Section. Here`s the beginning of the sequence :

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ..... .

Each number is the sum of the two preceding numbers. They satisfy

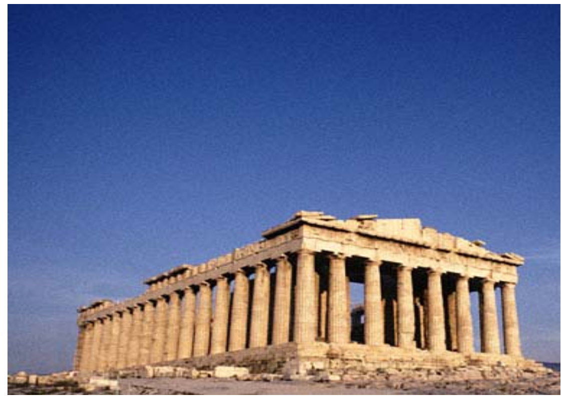
$$F(n+1)=F(n)+F(n-1).$$

### The Golden Ratio/Mean/Section

A special value, closely related to the Fibonacci series, is called the golden section (ratio, mean). This value is obtained by taking the ratio of successive terms in the Fibonacci series.

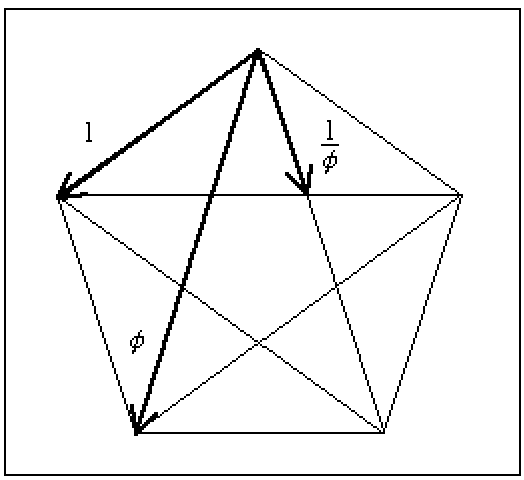
If you plot a graph of these values you'll see that they seem to be tending to a limit of (1+\sqrt(5))/2 approximately =1.618). This limit is actually the positive root of a quadratic equation and is called the golden section, golden ratio or sometimes the golden mean.

The golden section is normally denoted by the Greek letter phi. In fact, the Greek mathematicians of Plato's time (400BC) recognized it as a significant value and Greek architects used the ratio 1:phi as an integral part of their designs, the most famous of which is the Parthenon in Athens.



## Phi and geometry

Phi also occurs surprisingly often in geometry. For example, it is the ratio of the side of a regular pentagon to its diagonal. If we draw in all the diagonals then they each cut each other with the golden ratio too (see picture). The resulting pentagram describes a star which forms part of many of the flags of the world.

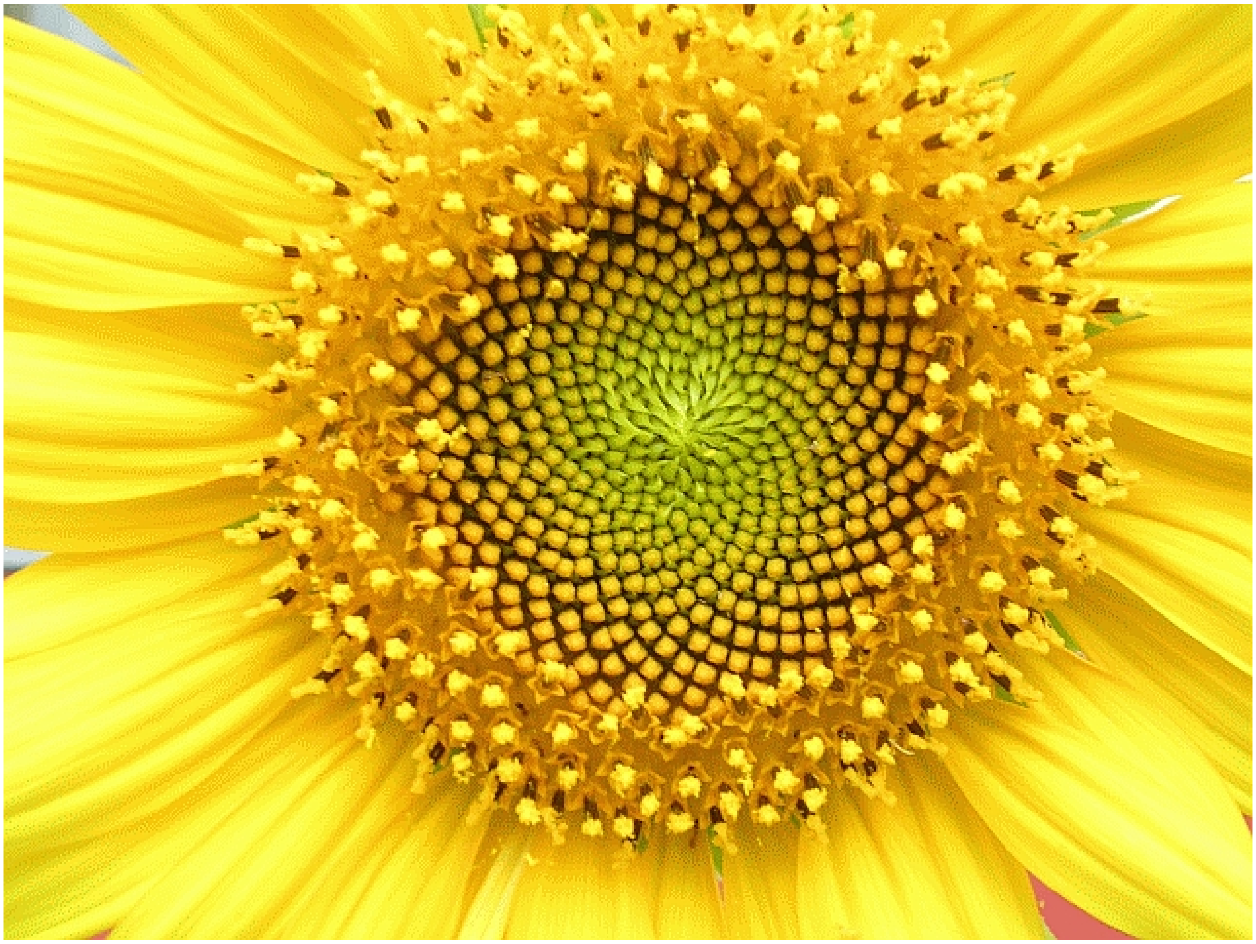



The pentagram star features in many of the world's flags, including the European Union and the United States of America.

## Fibonacci in nature

The rabbit breeding problem that caused Fibonacci to write about the sequence in Liber abaci may be unrealistic but the Fibonacci numbers really do appear in nature. For example, some plants branch in such a way that they always have a Fibonacci number of growing points.

Flowers often have a Fibonacci number of petals; daisies can have 34, 55 or even as many as 89 petals! Next time you look at a sunflower, take the trouble to look at the arrangement of the seeds. They appear to be spiraling outwards both to the left and the right. There are a Fibonacci number of spirals! The following sunflower has 34 left spirals and 55 right spirals.



The following sunflower pattern has 21 left spirals and 34 right spirals.