ASSIGNMENT 9

MAT 472 · FALL 2005

Also look at: Problems 13–20, 24–30, and 33–40 from Chapter IV of Rosenlicht.

Problem 1 (Problem IV.21)**.** Let S be a dense subset of a metric space E, let F be a complete metric space, and let $f\colon S \to F$ be uniformly continuous. Prove that there exists a unique function $g\colon E \to F$ such that

(i) g is continuous, and
(ii) $g(s) = f(s)$ for each $s \in S$.

(So g is a *continuous extension* of f to E.) Also prove that g is in fact uniformly continuous.

Problem 2 (See Problem IV.29)**.** A set S in a metric space E is *arcwise connected* if, for each $p, q \in S$, there exists a continuous function $f\colon [0,1] \to S$ such that $f(0) = p$ and $f(1) = q$.

(a) Show that every arcwise connected metric space is connected.
(b) Show that every connected open subset of E^n is arcwise connected.

Problem 3 (Problem IV.34)**.** Determine whether or not each sequence of functions converges uniformly on $[0,1]$. Justify your answers.

(a) $f_n(x) = \dfrac{x}{1+nx^2}$ (b) $g_n(x) = \dfrac{nx}{1+nx^2}$ (c) $h_n(x) = \dfrac{nx}{1+n^2x^2}$

Problem 4 (Problem IV.41)**.** Let (f_n) be a sequence of continuous real-valued functions on a compact metric space E which converges pointwise to a continuous function f. Prove that if $f_n(p) \le f_{n+1}(p)$ for each $p \in E$ and $n \in \mathbb{N}$, then $f_n \to f$ uniformly on E.

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