# ASSIGNMENT 6

MAT 472 · FALL 2005

**Problem 1.** Let $(E, d_E)$ and $(F, d_F)$ be separable metric spaces. Prove that $(E \times F, d)$ is separable, where $d$ is the metric on $E \times F$ defined by

$$d((e, f), (e', f')) = \max\{d_E(e, e'), d_F(f, f')\}.$$

A metric space $(E, d)$ is *totally bounded* if for each $\epsilon > 0$ there exists a finite set $F \subseteq E$ such that

$$E = \bigcup_{p \in F} B_\epsilon(p).$$

**Problem 2.** Prove that every totally bounded metric space is separable.

**Problem 3** (See Problem III.36)**.** Suppose $(E, d)$ is a metric space which is not totally bounded. Prove that there exists a sequence in $E$ which has no Cauchy subsequence.

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