ASSIGNMENT 2

MAT 472 · FALL 2005

Problem 1 (II.10c)**.** Find the inf and sup of the set

$$S = \left\{ \sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \dots \right\}.$$

Justify your answers.

Problem 2 (see III.1)**.** (b) Prove that (ℓ^∞, d) is a metric space, where ℓ^∞ is the set of all bounded infinite sequences of real numbers, and d is defined by

$$d((x_i),(y_i)) = \sup\{|x_i - y_i| \mid i \in \mathbb{N}\}.$$

(c) Suppose (E, d_E) and (F, d_F) are metric spaces. Prove that $(E \times F, d)$ is a metric space, where d is defined by

$$d((a,b),(x,y)) = \max\{d_E(a,x), d_F(b,y)\}.$$

(d) Sketch the open balls in the metric space of part (c), if we use $E = F = \mathbb{R}$, with the usual metric: $d_E(x,y) = d_F(x,y) = |x-y|$.

Problem 3 (III.4)**.** Show that the set $S = \{(x,y) \mid x > y\}$ is open in E^2.

Problem 4 (III.6)**.** Show that the set $S = \{(x,y) \mid xy = 1,\ x > 0\}$ is closed in E^2.

Problem 5 (III.5)**.** Prove that any bounded open subset of $\mathbb{R}$ (with the usual metric, $d(x,y) = |x-y|$) is the union of disjoint open intervals.

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