ASSIGNMENT 1

MAT 472 · FALL 2005

Problem 1. Let X and Y be nonempty sets, and let $f\colon X \to Y$ be a function. Recall that id_S denotes the identity function $s \mapsto s$ from a set S into itself.

(a) Prove that f is one-to-one if and only if there exists a function $g\colon Y \to X$ such that $g \circ f = \mathrm{id}_X$.

(b) Prove that f is onto Y if and only if there exists a function $h\colon Y \to X$ such that $f \circ h = \mathrm{id}_Y$.

(c) Prove that if there exist both g and h as in parts (a) and (b), then we must have $g = h$.

Problem 2. This exercise fills in some of the details of Proposition 1.5 in my notes, so do not use that proposition in your proofs. For $n \in \mathbb{N}$, let $S_n = \{1, 2, \ldots, n\}$.

(a) Prove that for any infinite set X, there exists a set $\{x_i \mid i \in \mathbb{N}\} \subseteq X$ such that $x_i \neq x_j$ for $i \neq j$.

(b) Given that, for a fixed $n \in \mathbb{N}$, there is no bijection of S_n onto a proper subset of S_n, prove that there is no bijection of S_{n+1} onto a proper subset of S_{n+1}.

(c) Given that for every $n \in \mathbb{N}$ there is no bijection of S_n onto a proper subset of S_n, prove that there is no bijection of X onto a proper subset of X for *any* finite set X.

Problem 3 (II.3)**.** Prove that if $a, b \in \mathbb{R}$ such that $a < b < 0$, then $1/a > 1/b$. Use references to facts from Rosenlicht (for example, "by **F7**") to justify each step of your argument.

Problem 4 (II.12)**.** Let X and Y be nonempty subsets of $\mathbb{R}$ such that $X \cup Y = \mathbb{R}$, and suppose that

$$x < y \quad \text{for all } x \in X \text{ and } y \in Y.$$

Prove that there exists $a \in \mathbb{R}$ such that either

$$X = \{x \in \mathbb{R} \mid x \leq a\} \quad \text{or} \quad X = \{x \in \mathbb{R} \mid x < a\}.$$

Problem 5 (II.13)**.** If S and T are nonempty subsets of $\mathbb{R}$ which are bounded above, prove that

$$\sup\{s + t \mid s \in S, t \in T\} = \sup S + \sup T.$$

Problem 6 (II.14)**.** Prove that for each $a, b \in \mathbb{R}$ with $a < b$, there exists an *irrational* number $x \in \mathbb{R}$ such that $a < x < b$.

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