1. (27) Give the definitions and statements of these important terms and theorems.

   (a) Order properties of $\mathbb{R}$. *Upper bound, bounded above, maximum, supremum*; *lower bound, bounded below, minimum, infimum*; *bounded sets.*

   (b) Least Upper Bound Prinicple (Completeness of $\mathbb{R}$). Archimedean Principle. Approximation of Suprema. Well-Ordering of $\mathbb{N}$. Density of $\mathbb{Q}$ in $\mathbb{R}$. Triangle Inequality in $\mathbb{R}$.

   (c) *Sequences. Convergent* and *divergent sequences.*

   (d) Convergent Sequences Are Bounded. Algebra of Limits for Sequences. Order Property of Limits for Sequences. Squeeze Theorem. Monotone Sequence Theorem.

   (e) *Subsequences.* Monotone Subsequence Theorem. Bolzano-Weierstrass Theorem I.

   (f) *Cauchy sequences.* Every Cauchy Sequence Converges. *Interior points, boundary points, isolated points, accumulation points. Interior of a set, boundary of a set.*

   (g) *Closed sets* and *open sets.* Characterization of Open Sets in $\mathbb{R}$. Relation between open-ness and closed-ness. Intersections and unions of open and closed sets.

   (h) *Compact sets.* Bolzano-Weierstrass Theorem II. *Countable, finite, infinite, countably infinite, uncountable.* Subsets and unions of countable sets. *Limit of a function at a point.* Sequential Characterization of Limits.

   (i) Uniqueness of Limits. Algebra of Limits of Functions.

   (j) Order Property of Limits of Functions. *Continuity at a point: ϵ-δ definition.* Limit Characterization of Continuity. Sequential Characterization of Continuity.

   (k) Algebra of Continuous Functions. Composition of Continuous Functions. *Continuity on a set. Uniform Continuity.*

   (l) Extreme Value Theorem. Intermediate Value Theorem.

   (m) *One-sided limits. Derivative* of a function at a point. *Differentiable functions.* Derivatives and continuity. Algebra of Derivatives.

   (n) Chain Rule. Critical Point Lemma. Rolle's Theorem and the Mean Value Theorem.

   (o) Cauchy's (Generalized) Mean Value Theorem. L'Hôpital's Rule.

   (p) Inverse Function Theorem. *Convex functions.* Convexity and derivatives.

   (q) *Taylor polynomials.* Lagrange's Remainder Theorem. *Partitions, Riemann sums.*

   (r) *Riemann integrability.* Integrable Functions Are Bounded. Continuous Functions Are Integrable. *"Improper" integrals.*

   (s) $m(\pi)$ and $M(\pi)$. Calculation of integrals using Riemann sums. Equivalent Characterizations of Integrability. Algebra of Integration. Order Properties of the Integral.

   (t) The Fundamental Theorems of Calculus.

   (u) *Series, partial sums, convergent series, divergent series.* Algebra of Series. *Geometric, harmonic, p-harmonic series.* Cauchy Criterion for Series.

   (v) *Absolute convergence, conditional convergence.* Absolute Convergence Implies Convergence. Comparison Test. Limit Comparison Test. Ratio Test. Root Test. Integral Test.

   (w) Alternating Series Test. *Pointwise convergence* of sequences of functions.

   (x) *Uniform convergence* of sequences of functions. Uniform Cauchy Criterion. Uniform Limits of Continuous Functions Are Continuous. Uniform convergence and integrals. Uniform convergence and derivatives.

   (y) *Power series. Radius of convergence.*

   (z) Uniform convergence of power series. Integration and differentiation of power series.

   (aa) *Power series representations. Taylor series.*

NAME: ______________________________

MAT 371 B

FINAL REVIEW

November 29, 2006

Instructions. This is a review sheet, not a sample exam. It consists of a day-by-day listing of the *most important* terms and theorems (not all of them) from the course. The final exam will be comprehensive. The format of the final will be similar to that of the midterms.

Here are some suggestions for studying:

1. Make sure you know solidly the definitions, basic facts, and statements of main theorems. Use flash cards, or make a detailed summary sheet, if it helps.
2. Make a list of main theorems and basic facts and use it to practice making and answering your own "example" questions. For example, the Intermediate Value Theorem applies to continuous functions. Is there an example of a non-continuous function for which the conclusion fails?
3. Use the non-turned-in homework problems to practice problem-solving: concentrate on understanding the problems, then sketching the main outline of the proof: which definitions and major theorems are likely to be involved? Can you make a picture of the situation? Carefully writing out detailed solutions of homework problems is not the best use of time.
4. De-construct my homework solutions: outline the basic definitions and facts involved, and the strategy of proof. Try to make a picture which illustrates the central ideas.

Problem	Points	Score
1	27	
Total	27	

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