1. (25) Give the definitions and statements of these important terms and theorems, and how to use them.

(a) $\mathbb{R}^n$. *Systems* of *linear equations* and their *solutions*. Finding all solutions for systems of linear equations.

(b) The *augmented matrix* of a linear system. *Elementary row operations*. *Row-equivalence* of matrices. *Reduced row-echelon form*. Row-reducing matrices. *Leading* and *free variables*. Gauss-Jordan elimination.

(c) *Matrix, column vector*. Algebra of Matrices.

(d) The *identity matrix*. *Invertible matrices*. Finding inverses.

(e) Equivalent Characterizations of Invertibility. *Determinants*.

(f) Calculating determinants. Determinants and elementary row operations. Determinants and invertibility. Determinants and products.

(g) *Vector spaces*.

(h) *Subspaces* of vector spaces. *Null space* and *column space* of a matrix.

(i) *Linear combinations, linear span, linear independence*.

(j) Too Few Vectors Can't Span; Too Many Vectors Can't Be Independent. Equivalent Characterizations of Independence for n Vectors in $\mathbb{R}^n$.

(k) *Basis* for a vector space. *Dimension*. Equivalent Characterizations of Independence for n Vectors in an n-dimensional Vector Space.

(l) *Coordinates*. Finding coordinates from vectors, and *vice versa*.

(m) *Transition matrix* for change of coordinates.

(n) Finding bases for column spaces and null spaces. *Rank* and *nullity* of a matrix. Rank-Nullity Theorem.

(o) *Linear transformations*. *Kernel* and *range* of a linear transformation.

(p) *Matrix representations* of a linear transformation.

(q) *Similar matrices*. Matrix representations and similarity.

(r) *Scalar product*, *length*, and *angle* in $\mathbb{R}^n$. Cauchy-Schwarz Inequality. *Perpendicular* vectors. *Vector* and *scalar projections*.

(s) *Orthogonal subspaces, orthogonal complements*. $\text{Null}(A^T) = \text{Col}(A)^\perp$. Finding $X^\perp$.

(t) *Least squares solutions, normal equations, orthogonal projections*.

(u) *Inner product spaces*. *Length* and *angle* in inner product spaces. *Orthogonal vectors*, *orthogonal subspaces*, *orthogonal complements*.

(v) *Orthonormal sets* and *bases*. Orthonormal bases and orthogonal projections. Orthonormal bases and coordinates.

(w) *Gram-Schmidt Orthonormalization*. Finding orthonormal bases.

(x) *Eigenvalues*, *eigenvectors*, and *eigenspaces*. *Characteristic polynomial*. Finding eigenthings.

(y) *Diagonalizable* matrices. Sufficient Conditions for Diagonalizability. Finding diagonalizing matrices.

NAME: ______________________________

MAT 342 F

FINAL REVIEW

November 29, 2006

Instructions. This is a review sheet, not a sample exam. It consists of a day-by-day listing of the *most important* terms and theorems (not all of them) from the course. The final exam will be comprehensive. The format of the final will be similar to that of the midterms.

Here are some suggestions for studying:

1. Make sure you know solidly the definitions, basic facts, and statements of main theorems. Use flash cards, or make a detailed summary sheet, if it helps.
2. Make a list of main theorems and basic facts and use it to practice making and answering your own "example" questions. For example, is there a matrix whose rank is 2 and whose nullity is 1? Where would you look for such a matrix?
3. Use the non-turned-in homework problems to practice problem-solving. Concentrate on understanding the problems, then sketching the main outline of the solution. What calculations must be made? Which definitions and major theorems are likely to be involved? Carefully writing out detailed solutions of homework problems is not the best use of time.
4. Understand my homework solutions. Outline the basic definitions and facts involved, and the strategy of of the solution or argument. If possible, make a picture which illustrates the central ideas.

Problem	Points	Score
1	25	
Total	25	

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