MAT 362 Advanced Math for Scientists and Engineers

Feb. 8, 2001 Test 1 name________________

Show your work - explain what you are doing. Computer printouts without explanations and formulas scattered over a page without clear logical order will be ignored (ZERO CREDIT!) It is YOUR responsibility to demonstrate that you have mastered the material of this class. Check ALL results with available computer software!

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1. Let f be the function with period π that is defined for all $t \in \mathbf{R}$ by
$$f(t) = \begin{cases} t & \text{if } 0 \le t \le \frac{\pi}{3} \\ \frac{\pi}{3} & \text{if } \frac{\pi}{3} \le t \le \frac{2\pi}{3} \\ \pi - t & \text{if } \frac{2\pi}{3} \le t \le \pi \end{cases}$$

a. Sketch the graph of f **over at least 2 periods**. (Label the graph!)
b. Is f even, or odd, or neither?
c. Calculate the (complex **or** real) Fourier coefficients of f.
d. Use your result from **c**. to write f as a Fourier series.
e. Give 4-digit-accuracy decimal approximations of the first 3 **nonzero** Fourier coefficients.
f. Use your result from **e**. to write out a 3-term Fourier approximation of f – call it F_3 in the sequel.
g. Overlay the plots of f and F_3.
h. Calculate the *mean square error* $E_3 = \int_0^\pi |f(t) - F_3(t)|^2\,dt$ (4-digit-accuracy decimal approximation).

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2. Suppose $f(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{ikt}$ (or $f(t) = a_0 + \sum_{k=-\infty}^{\infty} (a_k \cos(kt) + b_k \sin(kt)))$.
a. Derive the standard formulas for the Euler coefficients c_k (or the coefficients a_k and b_k) – it suffices to **precisely describe** the main steps and write formulas only where really needed.
Bonus: Suppose f is 2π periodic and c_k are the standard (complex) Fourier coefficients. Show that

$$\int_{\pi}^{\pi} |f(t) - F_N(t)|^2\,dt = 2\pi \sum_{k=N+1}^{\infty} |c_k|^2. \qquad \text{where } F_N(t) = \sum_{k=-N}^{N} c_k e^{ikt}$$

3.a. Does a function f have to be differentiable, or continuous in order to *have* a Fourier series?

b. Does the Fourier series *converge* to the original function at every point x? **Be specific!**
c. Explain what the Gibbs phenomenon is.
d. Explain how Bessel's inequality and Parseval's identity are related to the convergence of Fourier series.

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4. Consider the three functions f, g, and h defined by:
$f(2m + x) = |x - 1|$ for any integer m and $0 \le x \le 2$.
$g(x) = |x - 1|$ for $0 \le x \le 2$.
$h(x) = |x - 1|$ for $0 \le x \le 2$ and $h(x) = 0$ if $x < 0$ or $x > 2$.
a. Sketch the graphs of the three functions. **Clearly show the differences!**
b. Which of these functions can be written as a Fourier series?
c. For which of these functions does one have the freedom to express it as a cosine or as a sine series?
d. Which of these functions may be written as a Fourier integral?
Bonus: Write one of these functions as a Fourier integral.