MAT 362 Advanced Math for Scientists and Engineers
October 9, 2003 Test 1 name_____

Explain what you are doing. Computer printouts without explanations, and scattered formulas without clear logical order will be ignored (zero credit!) It is YOUR responsibility to demonstrate that you have mastered the course material. Check ALL results with available computer software!

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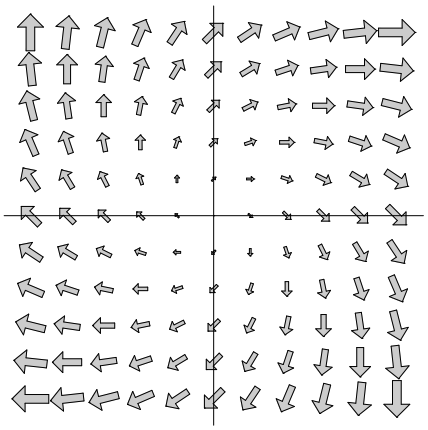
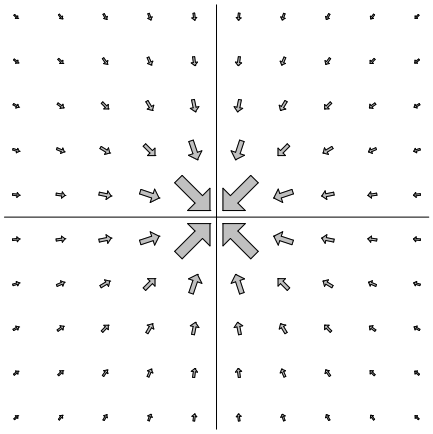
1. Evaluate the line integrals $\int_C \vec{F} \cdot d\vec{R}$ for each of the following pairs of vector fields and curves. Demonstrate your knowledge by using, if possible, a different approach in every case – strive for elegance and diversity. Credit will be given for (i) correct value, (ii) demonstration of many different approaches (once each suffices), and (iii) clarity of explanations for all major steps (i.e. those beyond simple calculus.)

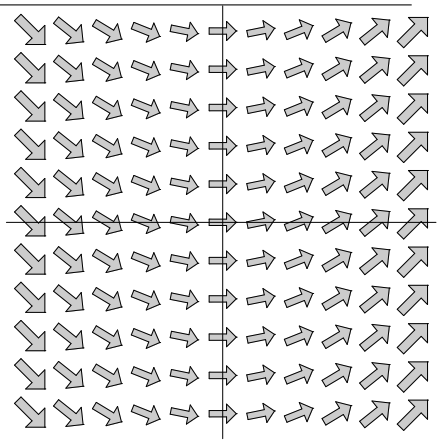
a. $\vec{F}(x,y) = y\vec{\imath} + x\vec{\jmath}$, and C is the line segment from $(-4,-2)$ to $(6,8)$.
b. $\vec{F}(x,y) = y^2\vec{\imath}$, and C is the circle with center $(0,0)$ and radius 3 traversed counter-clockwise.
c. $\vec{F}(x,y) = x\vec{\imath} + y\vec{\jmath}$; and C is the same circle.
d. $\vec{F}(x,y) = y^3\vec{\imath} - x^3\vec{\jmath}$, and C is the same circle.
e. $\vec{F}(x,y) = 2y\vec{\imath} + 3x^2\vec{\jmath}$, and C is the part of the parabola $y = x^2$ from $(2,4)$ to $(3,9)$.
f. $\vec{F}(x,y) = 2x\vec{\imath} + 3y^2\vec{\jmath}$, and C is the same curve as in **e.**



$\vec{F}(x,y)$



$\vec{G}(x,y)$



$\vec{H}(x,y)$

2.a. Which of the three vector fields appear to be linear?
b. Which of the vector fields shown appear to be conservative? Explain!
c. Which of the vector fields shown appear to be divergence free? Explain!
d. Find a possible formula for each vector fields.
Hint: For the 3rd field think of a constant drift added to a simple vector field – look at $\vec{H}(0,0)$.
e. If possible, find a *potential function* for each vector field. If impossible, **explain why**.

3.a. In which physical setting does the vector field $\vec{F}(x,y) = \dfrac{-y}{x^2+y^2}\,\vec{\imath} + \dfrac{x}{x^2+y^2}\,\vec{\jmath}$ arise?
b. Calculate the *"scalar curl"* $\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$ of the vector field $\vec{F}$. **Show details** of your calculation.
c. Let C_1 be the polygonal curve with vertices $(7,1)$, $(2,6)$, $(-3,-1)$, $(1,3)$ and $(3,-1)$ oriented counterclockwise. Evaluate $\oint_{C_1} \vec{F} \cdot d\vec{R}$.
d. Directly evaluate $\oint_{C_2} \vec{F} \cdot d\vec{R}$ where C_2 is the unit circle $x^2 + y^2 = 1$.
e. Let C_3 be the polygonal curve with vertices $(7,1)$, $(2,6)$, $(-3,-1)$, $(1,-3)$ and $(3,-1)$ traversed counterclockwise. **Explain in detail** how to use Green's theorem and the result from **d**. to