MAT 571 **HW 6** **Due Thursday, 2/29/07**

Solve three problems from among these and past unsolved problems.

23. Let X be a TVS, and let $Y \subseteq X$ be a finite dimensional linear manifold. Prove that Y is closed.

24. Let X be an infinite dimensional normed space. Prove that X is not locally compact.

25. Recall the following sequence spaces:

c_0 is the space of all complex sequences converging to 0.
c is the space of all convergent complex sequences.
ℓ^1 is the space of all absolutely summable complex sequences.

Let c_0 and c have the infinity norm ($\|(x_i)\|_\infty = \sup_i |x_i|$), and let ℓ^1 have the 1-norm ($\|(x_i)\|_1 = \sum_i |x_i|$). Then c_0, c, and ℓ^1 are Banach spaces.

(i) Prove that $(c_0)^* = \ell^1$ via the pairing $c_0 \times \ell^1 \to \mathbf{C}$ given by $\langle x, y\rangle = \sum_i x_i y_i$.
(ii) Prove that c^* is isomorphic to ℓ^1.
(iii) Are c_0 and c isomorphic? Prove your answer.

26. (i) Let X and Y be normed spaces, let $M \subseteq X$ be a closed subspace, let $S \in B(X/M, Y)$, and let $\pi : X \to X/M$ be the quotient map. Prove that $\|S \circ \pi\| = \|S\|$.

(ii) Let $T \in B(X, Y)$ and let $N = \ker T$. Prove that there is a unique linear map $T_0 : X/N \to Y$ such that $T_0 \circ \pi = T$, that this map is bounded, and that $\|T_0\| = \|T\|$.