Solve three problems from among these and past unsolved problems.

19. Let $f : [0, \infty) \to \mathbf{C}$ be a bounded measurable function. Suppose that

$$\int_0^\infty f(x) e^{-nx} \, dx = 0$$

for $n = 1, 2, 3, \ldots$. Prove that $f = 0$ almost everywhere.

20. (Note that the functions in this problem are complex-valued.) Let X and Y be locally compact Hausdorff spaces. Let $\mathcal{F}$ be the collection of functions f of the form $f(x, y) = g(x)h(y)$, where $g \in C_0(X)$ and $h \in C_0(Y)$.
(i) Prove that $\mathcal{F} \subseteq C_0(X \times Y)$.
(ii) Prove that $\operatorname{span} \mathcal{F}$ is uniformly dense in $C_0(X \times Y)$.

21. Let X be a TVS and let $M \subseteq X$ be a linear manifold. Let X/M have the quotient topology. (Recall that this is the topology $\{A \subseteq X/M : \pi^{-1}(A) \text{ is open in } X\}$, where $\pi : X \to X/M$ is the quotient map.)
(i) Prove that addition and scalar multiplication are continuous on X/M.
(ii) Prove that X/M is Hausdorff (and hence a TVS) if and only if M is closed in X.

22. Let X be a TVS, and let $f : X \to \mathbf{C}$ be a non-zero linear functional. Prove that the following statements are equivalent.
(i) f is continuous.
(ii) $\ker f$ is closed.
(iii) $\ker f$ is not dense in X.