MAT 571 **HW 4** **Due Thursday, 2/15/07**

Solve three problems from among these and past unsolved problems.

15. Let X be an LCH space, and let $E \subseteq X$.
(i) Prove that if E is open then the relative topology of E is locally compact.
(ii) Prove that if E is dense in X, and if the relative topology of E is locally compact, then E is open. (Problem 2 will be useful.)
(iii) Prove that the relative topology of E is locally compact if and only if E is relatively open in $\overline{E}$.

16. Let X be an LCH space. Let $K \subseteq X$ be compact, let $U \subseteq \mathbf{C}$ be open, let $f \in C(X)$, and let $\epsilon > 0$. Define two subsets of $C(X)$ as follows:

$$V(f, K, \epsilon) = \big\{ g \in C(X) : \|f - g\|_K < \epsilon \big\}$$

$$W(K, U) = \big\{ g \in C(X) : g(K) \subseteq U \big\},$$

where $\|h\|_K = \sup\big\{|h(x)| : x \in K\big\}$. Let $\mathcal{T}$ be the topology (on $C(X)$) generated by all sets of the form $V(f, K, \epsilon)$, and let $\mathcal{T}'$ be the topology generated by all sets of the form $W(K, U)$. Prove that $\mathcal{T} = \mathcal{T}'$.

(This topology on $C(X)$ is called the *compact-open topology*, or the *topology of uniform convergence on compact sets.*)

17. A continuous function f is called *proper* if $f^{-1}(K)$ is compact whenever K is compact. Let $f : X \to Y$ be a continuous map between LCH spaces. Prove that f is proper if and only if f extends to a continuous map $\widetilde{f} : \widetilde{X} \to \widetilde{Y}$ by setting $\widetilde{f}(\infty_X) = \infty_Y$.

18. Let X be an LCH space, and let $A \subseteq C(X)$. Suppose that A is compact for the compact-open topology. Prove that A is pointwise bounded and equicontinuous.