MAT 571 **HW 2** **Due Thursday, 2/1/07**

Solve three problems from among these and past unsolved problems.

5. Let Y be a set, let $\{X_i\}_{i\in I}$ be a family of Hausdorff topological spaces, and let $f_i : Y \to X_i$ be functions. Let $\mathcal{T}$ be the weak topology defined by the $\{f_i\}$. Prove that $\mathcal{T}$ is Hausdorff if and only if for all $y \neq w$ in Y there exists $i \in I$ such that $f_i(y) \neq f_i(w)$.

6. Let X be a topological space, Y a Hausdorff space, and $f, g : X \to Y$ continuous maps.
(i) $\{x : f(x) = g(x)\}$ is closed.
(ii) If $f = g$ on a dense subset of X, then $f = g$ on all of X.

7. Let X be a topological space. A subset of X is called a G_δ if it can be written as the intersection of countably many open subsets of X.
(i) Let $f : X \to \mathbf{R}$ be continuous. Prove that $f^{-1}(\{0\})$ is a closed G_δ.
(ii) Suppose that X is normal, and let A be a closed G_δ subset of X. Prove that there is a continuous function $f : X \to \mathbf{R}$ such that $A = f^{-1}(\{0\})$.

8. A topological space Y is called an *absolute retract* if the following holds: whenever X is a normal space, $A \subseteq X$ is a closed subset, and $f : A \to Y$ is a continuous map, then there is a continuous map $F : X \to Y$ such that $F|A = f$ (i.e. every continuous map from A to Y can be extended to a continuous map from X to Y).
(i) Let $A \subset \mathbf{R}$. Prove that A is an absolute retract if and only if A is an interval.
(ii) Let $\{Y_i\}_{i\in I}$ be a family of absolute retracts. Let $Y = \prod_{i\in I} Y_i$ have the product topology. Prove that Y is an absolute retract.