

Solve three problems from among these and past unsolved problems.

46. Let X be a locally compact Hausdorff space. Let $\varphi : C_0(X) \rightarrow \mathbf{C}$ be a positive linear functional. Prove that φ is bounded. (Hint: suppose not, and construct a bounded sequence in $C_0(X)_+$ on which φ tends rapidly to ∞ .)

47. Let X be a locally compact Hausdorff space. Prove that a sequence f_i in $C_0(X)$ converges weakly to $f \in C_0(X)$ if and only if $\sup_i \|f_i\|_u < \infty$ and $f_i \rightarrow f$ pointwise.

48. Let X be a locally compact Hausdorff space, and let μ be a Radon measure on X . Let $\phi \in L^1(\mu)$, and define a Borel measure ν on X by $\nu(E) = \int_E \phi d\mu$. Prove that ν is a Radon measure on X .