MAT 571 **HW 1** **Due Thursday, 1/25/07**

Solve three problems.

1. Prove that a separable metric space is second countable.

2. Let X be a topological space, let U be an open subset, and let A be a dense subset. Prove that $\overline{U} = \overline{U \cap A}$.

3. Let $\mathcal{B} = \big\{(a, b] : -\infty < a < b < \infty\big\}$.

(i) $\mathcal{B}$ is a base for a topology $\mathcal{T}$ on $\mathbf{R}$ in which the members of $\mathcal{B}$ are both open and closed.

(ii) $\mathcal{T}$ is first countable but not second countable.

(iii) $\mathbf{Q}$ is dense in $\mathbf{R}$ with respect to $\mathcal{T}$.

(Thus separability and first countability do not imply second countability.)

4. Recall the set $X = \big\{f : [0, 1] \to [0, 1]\big\}$. For a, $b \in [0, 1]$ and $\epsilon > 0$ let

$$V(a, b, \epsilon) = \big\{f \in X : \big|f(a) - b\big| < \epsilon\big\},$$

and let the collection of all $V(a, b, \epsilon)$ generate a topology on X (it is called *the point-open topology*). Let A be the set of those f in X that are nonzero at only finitely many points.

(i) Prove that $1 \in \overline{A}$.

(ii) Prove that there does not exist a sequence in A converging to 1.