Mat 300, Spielberg **Test 2 — Review Problems** **Spring 07**

1. Define a relation R from $\mathbf{N}$ to $\mathbf{N}$ by:

$$R = \{(a,b) \in \mathbf{N} \times \mathbf{N} : \text{there exists } n \text{ in } \mathbf{Z} \text{ such that } a = b \cdot 2^n\}.$$

(i) Prove that R is an equivalence relation on $\mathbf{N}$.

(ii) What is the equivalence class of 3?

2. Let $x \in \mathbf{Z}$. Suppose that $x^2 \equiv_5 4$. Prove that $x \equiv_5 2$ or $x \equiv_5 -2$.

3. Prove that if $a|b$ and $a|c$ then for all m, $n \in \mathbf{Z}$, $a|(mb + nc)$.

4. Let $a \equiv_6 4$ and $b \equiv_6 3$. Prove that $a^2 + b^2 \equiv_6 1$.

5. Prove that if $n \in \mathbf{Z}$ is odd then $8|(n^2 - 1)$.

6. Prove that for all integers a, 3 does not divide $a^2 + 1$. (Hint: consider the possible remainders when a is divided by 3.)

7. Problems F, # 3, 5(ii), 7.

8. Let M be the following relation on $\mathbf{R}$: $M = \{(x,y) \in \mathbf{R} \times \mathbf{R} \; : \; |x - y| < 1\}$. Is M reflexive? symmetric? antisymmetric? transitive? Prove your answers.

9. Let $f : A \to B$ be a one-to-one function. Let S_1, $S_2 \subseteq A$, and suppose that $f(S_1) \subseteq f(S_2)$. Prove that $S_1 \subseteq S_2$.

10. Problems G, # 2(i,iii), 3(ii,iii), 4(ii,iii), 5(iii)