Mat 300, Spielberg **Test 1 Review Problems** **Spring 07**

1. (i) Show that the formulas $(p \vee q) \to r$ and $(p \to r) \wedge (q \to r)$ are logically equivalent. Do this by using known equivalences (such as commutativity, associativity, distributivity, and DeMorgan's laws) to modify one of the formulas until it becomes the other. Show all steps in the process.

(ii) Do the same for $p \to (q \to r)$ and $(p \wedge q) \to r$.

2a. Rewrite the following statements symbolically without using union, intersection, containment, or negation symbols (but you may use logical symbols in this problem):

$$x \in \bigcap_{i \in I} A_i$$

$$x \notin \bigcap_{i \in I} A_i$$

2b. Rewrite the following statements in equivalent form using only the symbols: A, B, C, $\cup$, $\cap$, $\subseteq$, (,), $=$, $\neq$, $\emptyset$. Show all intermediate steps.

$$A \setminus B \subseteq C$$

$$A \setminus B \not\subseteq C$$

3. Prove that $A \setminus \bigcap_{i \in I} B_i = \bigcup_{i \in I} (A \setminus B_i)$.

4. Let A, B, and C be sets. Suppose that $A \subseteq B' \cup C$. Prove that $A \cap B \subseteq C$.

5. Let $\{A_i : i \in I\}$ and $\{B_j : j \in J\}$ be indexed families of sets. Suppose that for all $i \in I$ and for all $j \in J$ we have $A_i \subseteq B_j$. Prove that

$$\bigcup_{i \in I} A_i \subseteq \bigcap_{j \in J} B_j.$$

6. Use axioms (A11) and (A12) to prove that the multiplicative identity in $\mathbf{R}$ is unique.

7. Use axioms (A10), (A11), (A12) and (A13) to prove that the multiplicative inverse of a non-zero real number is unique.

8. Use properties 9b and 9h on page 59 to prove that for any real numbers a and b, if $a < b < 0$ then $a^2 > b^2$.

9. Suppose that $|x + 10| < 5$. Prove that $|x| > 5$.

10. (i) Prove that for every natural number n,

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{1}{3} n(n+1)(n+2).$$

(ii) Prove that for every natural number $n \geq 4$,

$$20n < 3^n.$$

11. Problems 1 and 2 on the sheet *Homework Problems C*, and Quiz 4.