MAT 571 **HW 6** **Due Thursday, 3/2/06**

Solve three problems. from among these and past unsolved problems.

22. Let X be a locally compact Hausdorff space. Recall that for $K \subseteq X$ compact, $U \subseteq X$ open, $f \in C(X)$, and $\epsilon > 0$, we define subsets of $C(X)$ by:

$$V(f, K, \epsilon) = \bigl\{g \in C(X) : \sup_{x \in K} \bigl|g(x) - f(x)\bigr| < \epsilon\bigr\}$$
$$W(K, U) = \bigl\{g \in C(X) : g(K) \subseteq U\bigr\}.$$

(The collection of all $V(f, K, \epsilon)$ is a subbase for the topology of uniform convergence on compact sets, while the collection of all $W(K, U)$ is a subbase for the compact-open topology.) It was proved in class that the compact-open topology is coarser than the topology of uniform convergence on compact sets. Prove that the two topologies are equal.

23. Let X be a locally compact Hausdorff space, and let $\mathcal{F} \subseteq C(X)$. Suppose that $\mathcal{F}$ is equicontinuous. Prove that $\overline{\mathcal{F}}$ is equicontinuous.

24. Let $K \in C\bigl([0, 1]^2\bigr)$. Define an operator $T : C[0, 1] \to C[0, 1]$ by

$$(Tf)(x) = \int_0^1 K(x, y) f(y)\, dy.$$

Prove that the image under T of the unit ball of $C[0, 1]$ is precompact. (The topology on $C[0, 1]$ is induced from the uniform norm.)

25. Let X and Y be compact Hausdorff spaces. Prove that every function in $C(X \times Y)$ can be uniformly approximated by a function of the form $g_1(x)h_1(y) + \cdots + g_k(x)h_k(y)$, where $g_i \in C(X)$ and $h_i \in C(Y)$.