MAT 571 **HW 4** **Due Thursday, 2/16/06**

Solve three problems. from among these and past unsolved problems.

13. Let $(X, \mathcal{T})$ be a compact Hausdorff space, and let $\mathcal{T}'$ be another topology on X.
(i) Prove that if $\mathcal{T}'$ is strictly finer than $\mathcal{T}$ then $(X, \mathcal{T}')$ is Hausdorff but not compact.
(ii) Prove that if $\mathcal{T}'$ is strictly coarser than $\mathcal{T}$ then $(X, \mathcal{T}')$ is compact but not Hausdorff.

14. Recall that each number in $[0, 1)$ has a base-2 expansion as a(n infinite) sum of inverse powers of 2, with coefficients in $\{0, 1\}$. The expansion is unique unless the number is a dyadic rational (i.e. a rational whose denominator is a power of 2). If we adopt the convention that for a dyadic rational we choose the finite expansion, then the expansion is unique. Then the nth coefficient defines a function $a_n : [0, 1) \to \{0, 1\}$. The functions a_1, a_2, . . . can be characterized by the equation

$$x = \sum_{n=1}^{\infty} a_n(x) 2^{-n}, \ x \in [0, 1).$$

Prove that the sequence (a_n) in the space $\{0, 1\}^{[0,1)}$ has no pointwise-convergent subsequence. (Thus the product topology on $\{0, 1\}^{[0,1)}$ is not sequentially compact.)

15. Let X be a locally compact Hausdorff space, let $K \subseteq X$ be a compact subset, and let $f : K \to \mathbf{R}$ be a continuous function. Prove that there is a continuous function $g : X \to \mathbf{R}$ such that $g\big|_K = f$ and such that $g = 0$ on the complement of some compact subset of X.

16. Let X be a locally compact Hausdorff space, and let $E \subseteq X$.
(i) Prove that if E is open then the relative topology of E is locally compact.
(ii) Prove that if E is dense in X, and if the relative topology of E is locally compact, then E is open. (Problem 2 will be useful.)
(iii) Prove that the relative topology of E is locally compact if and only if E is relatively open in $\overline{E}$.