MAT 571 **HW 3** **Due Thursday, 2/9/06**

Solve three problems. from among these and past unsolved problems.

9. Let X be a topological space. A subset of X is called a G_δ if it can be written as the intersection of countably many open subsets of X.

(i) Let $f : X \to \mathbf{R}$ be continuous. Prove that $f^{-1}(\{0\})$ is a closed G_δ.

(ii) Suppose that X is normal, and let A be a closed G_δ subset of X. Prove that there is a continuous function $f : X \to \mathbf{R}$ such that $A = f^{-1}(\{0\})$.

10. Let X and $\{Y_i\}_{i\in I}$ be topological spaces, and for $i \in I$ let $f_i : X \to Y_i$ be a continuous function. Let $Y = \prod_{i\in I} Y_i$, and define $f : X \to Y$ by $f(x)_i = f_i(x)$. Prove the following:

(i) f is continuous.

(ii) f is one-to-one if and only if the family $\{f_i\}_{i\in I}$ *separates points* of X (i.e. if $x \neq z$ in X then there exists $i \in I$ such that $f_i(x) \neq f_i(z)$.)

(iii) $f : X \to f(X)$ is an open map if whenever $x \in X$, and $E \subseteq X$ is closed with $x \notin E$, there is $i \in I$ such that $f_i(x) \notin \overline{f_i(E)}$. Here $f(X)$ has the relative topology inherited from Y. (A map between topological spaces is *open* if the image of every open set is open.)

11. Let X be a topological space. Prove that X is Hausdorff if and only if every net in X converges to at most one point.

12. Let X have the weak topology induced by a family $\mathcal{F}$ of functions on X. Let $x \in X$ and let $(x_a)_{a\in A}$ be a net in X. Prove that $x_a \to x$ if and only if $f(x_a) \to f(x)$ for each $f \in \mathcal{F}$.