MAT 571 **HW 11** **Due Thursday, 4/13/06**

Solve three problems. from among these and past unsolved problems.

44. Let X be a locally compact Hausdorff space. Let $\varphi : C_0(X) \to \mathbf{C}$ be a positive linear functional. Prove that φ is bounded. (Hint: suppose not, and construct a bounded sequence in $C_0(X)_+$ on which φ tends rapidly to ∞.)

45. Let X be a locally compact Hausdorff space. Prove that a sequence f_i in $C_0(X)$ converges weakly to $f \in C_0(X)$ if and only if $\sup_i \|f\|_u < \infty$ and $f_i \to f$ pointwise.

The following three problems involve the σ-algebra of Baire sets. Let X be a locally compact Hausdorff space. The *Baire sets* in X, $\mathcal{B}_X^0$, are the members of the smallest σ-algebra with respect to which all functions in $C_c(X)$ are measurable.

46. Let $\mathcal{E}$ be the collection of all compact G_δ subsets of X. Prove that $\mathcal{B}_X^0 = \mathcal{M}(\mathcal{E})$. (Hint: use problem 9.)

47. Let X be a second countable locally compact Hausdorff space.
(i) Prove that every compact subset of X is a compact G_δ set.
(ii) Prove that $\mathcal{B}_X = \mathcal{B}_X^0$.

48. Let $X = \mathbf{R}_d$, the real numbers with the discrete topology. Prove that $\mathcal{B}_X \neq \mathcal{B}_X^0$.