MAT 571 **HW 1** **Due Thursday, 1/26/06**

Solve three problems.

1. Prove that a separable metric space is second countable.

2. Let X be a topological space, let U be an open subset, and let A be a dense subset. Prove that $\overline{U} = \overline{U \cap A}$.

3. Let $\mathcal{B} = \{(a, b] : -\infty < a < b < \infty\}$.
(i) $\mathcal{B}$ is a base for a topology $\mathcal{T}$ on $\mathbf{R}$ in which the members of $\mathcal{B}$ are both open and closed.
(ii) $\mathcal{T}$ is first countable but not second countable.
(iii) $\mathbf{Q}$ is dense in $\mathbf{R}$ with respect to $\mathcal{T}$.

(Thus separability and first countability do not imply second countability.)

4. Let X be a topological space, Y a Hausdorff space, and f, $g : X \to Y$ continuous maps.
(i) $\{x : f(x) = g(x)\}$ is closed.
(ii) If $f = g$ on a dense subset of X, then $f = g$ on all of X.