Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions.

33. Let $f : \mathbf{R}^n \to \overline{\mathbf{R}}$ be measurable.

(i) Suppose that $f \geq 0$. Prove that $\int f = 0$ if and only if $f = 0$ a.e.

(ii) Suppose that $f = 0$ almost everywhere. Prove that f is integrable, and that $\int f = 0$.

(iii) Suppose that f is integrable. Prove that the set $\{x \in \mathbf{R}^n \mid f(x) \notin \mathbf{R}\}$ is a nullset.

34. Prove that the following variations on the statement of Fatou's lemma do not hold in general.

(i) Replace $\leq$ by $=$.

(ii) Replace lim inf by lim sup.

(iii) Replace lim inf by lim sup and replace $\leq$ by $\geq$.

35. Let $f : \mathbf{R}^n \to \overline{\mathbf{R}}$ be integrable. Let $\epsilon > 0$. Prove that there exists a measurable set E with $\lambda(E) < \infty$ and such that $\int |f| \chi_{E^c} < \epsilon$.

36. Let (X, d) be a metric space, let $f : X \to \mathbf{R}$ a function, and let $a \in X$. The *oscillation of f at a* is defined by

$$\operatorname{osc}(f, a) = \lim_{r \to 0} \Big(\sup_{x, y \in B_r(a)} \big| f(x) - f(y) \big| \Big).$$

(Note: the sup decreases as r decreases, so the limit exists.)

(i) Prove that f is continuous at a if and only if $\operatorname{osc}(f, a) = 0$.

(ii) Let $c > 0$. Prove that $\{x \in X \mid \operatorname{osc}(f, x) \geq c\}$ is a closed set.