MAT 473 **HOMEWORK 6** **Due: 3/11/05**

Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions.

21. Let $B \subseteq \mathbf{R}$ with $\lambda^*(B) > 0$. Prove that there exists $C \subseteq B$ such that $\lambda_*(C) = 0$ and $\lambda^*(C) > 0$. (In particular, every measurable set of positive measure contains a non-measurable subset.) (Hint: Let A be the non-measurable set constructed in class. Cover B with rational translates of A.)

22. Let $A \in \mathcal{L}_0$ and let $\epsilon > 0$. Prove that there are finitely many disjoint nondegenerate special rectangles $E_1, \ldots, E_k$ such that

$$\lambda\big(A \triangle (E_1 \cup \cdots \cup E_k)\big) < \epsilon.$$

(Recall that the *symmetric difference* of two sets is defined by $A \triangle B = (A \setminus B) \cup (B \setminus A)$).

23. (i) (continuity from above) Let $A_1, A_2, \ldots \in \mathcal{L}$ with $A_1 \supseteq A_2 \supseteq \cdots$. Suppose also that $\lambda(A_1) < \infty$. Prove that $\lambda(\cap_{i=1}^{\infty} A_i) = \lim_{i \to \infty} \lambda(A_i)$.

(ii) Show by means of an example that the statement in (i) is false if the finiteness hypothesis is omitted.

24. Let $A_1, A_2, \ldots \in \mathcal{L}$.

(i) Prove that $\lambda(\liminf A_i) \le \liminf \lambda(A_i)$.

(ii) Suppose that $\lambda(\cup_{i=1}^{\infty} A_i) < \infty$. Prove that $\lambda(\limsup A_i) \ge \limsup \lambda(A_i)$.

(Recall that the *liminf* and *limsup* of a sequence of sets are defined by

$$\liminf A_i = \cup_{i=1}^{\infty} \cap_{j=i}^{\infty} A_j$$

$$\limsup A_i = \cap_{i=1}^{\infty} \cup_{j=i}^{\infty} A_j.)$$