MAT 473 **HOMEWORK 11** **Due: 4/22/05**

Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions.

41. (i) Compute

$$\int_{\mathbf{R}^+\times\mathbf{R}^+} xe^{-x^2(1+y^2)}\,dxdy,$$

justifying all steps.

(ii) Show that

$$\int_0^\infty e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2},$$

justifying all steps.

42. (i) Let $a > 0$. Show that $\sin x e^{-xy}$ is integrable on $(0, a) \times (0, \infty)$, justifying all steps.

(ii) Prove that the following limits both equal zero:

$$\lim_{a\to\infty}\int_0^\infty \frac{1}{1+y^2}e^{-ay}\,dy \qquad \text{and} \qquad \lim_{a\to\infty}\int_0^\infty \frac{y}{1+y^2}e^{-ay}\,dy.$$

43. (Continuing problem 42.)

(i) Prove that

$$\int_0^a \frac{\sin x}{x}\,dx = \frac{\pi}{2} - \cos a\int_0^\infty e^{-ay}\frac{1}{1+y^2}\,dy - \sin a\int_0^\infty e^{-ay}\frac{y}{1+y^2}\,dy.$$

(ii) Prove that

$$\lim_{a\to\infty}\int_0^a \frac{\sin x}{x}\,dx = \frac{\pi}{2}.$$

44. Let $I \subseteq \mathbf{R}$ be an interval of length 7. Let $A \subseteq I$ be a measurable set with $\lambda(A) > 4$. Prove that $A \cap (A + 1)$ has positive measure.