MAT 372 **HOMEWORK 9** **Due: 4/15/05**

Please write **neatly** and **legibly**, write on **only one side of the paper**, print your name, and STAPLE the pages together before coming to class. Always show your work.

38. (i) Let $f : \mathbf{R}^n \to \mathbf{R}$, and let $f_\pm$ be the positive and negative parts of f. Let E be the set of discontinuities of f, and let $E_\pm$ be the set of discontinuities of $f_\pm$. Prove that $E = E_+ \cup E_-$.

(ii) Recall that $f : \mathbf{R}^n \to \mathbf{R}$ is called *admissible* if f is bounded, f has bounded support, and the set of discontinuities of f is negligible. Prove that f is admissible if and only if both f_+ and f_- are admissible.

39. (i) Let A, $B \subseteq \mathbf{R}^n$. Prove that $\partial(A \cup B)$, $\partial(A \cap B)$ and $\partial(A \setminus B)$ are all subsets of $\partial A \cup \partial B$.

(ii) Let A, $B \subseteq \mathbf{R}^n$ be bounded sets having content. Suppose that $A \cap B$ is negligible. Prove that $A \cup B$ has content, and that $v(A \cup B) = v(A) + v(B)$. (Use part (i), the theorem that a bounded set has content if and only if its boundary is negligible, and the definition of content.)