MAT 372 **HOMEWORK 7** **Due: 3/25/05**

Please write **neatly** and **legibly**, write on **only one side of the paper**, print your name, and STAPLE the pages together before coming to class. Always show your work.

30. (A version of the mean value theorem in $\mathbf{R}^n$.) Let $f : \mathbf{R}^n \to \mathbf{R}^m$ be continuously differentiable. Let x, $y \in \mathbf{R}^n$. Prove that there exists z on the open segment from x to y such that

$$\big|f(y) - f(x)\big| \leq \big|f'(z)\,(y - x)\big|.$$

(Hint: let $v = f(y) - f(x)$. Define $\phi : [0, 1] \to \mathbf{R}$ by $\phi(t) = \big[f\big((1-t)x + ty\big) - f(x)\big] \cdot v$.)

31. Prove that there exists a function $f : \mathbf{R} \to \mathbf{R}$ such that $\big|f(x) - f(y)\big| < |x - y|$ for all x, $y \in \mathbf{R}$ with $x \neq y$, and such that $f(x) \neq x$ for all $x \in \mathbf{R}$.

For the next two problems you will want to consult the *Notes on Matrix Norms,* available on the course webpage.

32. Find the operator norm and the 2-norm of the matrices.

(i) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ & & \dots & \\ 1 & 1 & \dots & 1 \end{pmatrix}_{n \times n}$

(Answers for first matrix: $\sqrt{15 + \sqrt{221}}$ and $\sqrt{30}$.)

33. Let $S \in M_n$ be invertible. Prove that if $T \in M_n$ with

$$\|T - S\| < \frac{1}{2\|S^{-1}\|}$$

then $\|T^{-1} - S^{-1}\| < 2\|S^{-1}\|^2\|T - S\|$. (This shows that inversion is continuous on the (open) set of invertible $n \times n$ matrices.)