Please write **neatly** and **legibly**, write on **only one side of the paper**, print your name, and STAPLE the pages together before coming to class. Always show your work.

**11.** Define $s$, $p : \mathbf{R}^2 \to \mathbf{R}$ by $s(x) = x_1 + x_2$ and $p(x) = x_1 x_2$. Prove that $s$ and $p$ are continuous.

**12.** Let $f : \mathbf{R}^n \to \mathbf{R}^m$.
(i) Suppose that for every open set $U \subseteq \mathbf{R}^m$ we have $f^{-1}(U)$ is open in $\mathbf{R}^n$. Prove that $f$ is continuous.
(ii) Suppose that for every closed set $E \subseteq \mathbf{R}^m$ we have $f^{-1}(E)$ is closed in $\mathbf{R}^n$. Prove that $f$ is continuous.

**13.** Let $(x_j)_{j=1}^{\infty}$ be a sequence in $\mathbf{R}^n$. Suppose that $|x_j| \geq j$ for all $j$. Prove that $(x_j)_{j=1}^{\infty}$ does not have a convergent subsequence.

**14.** Let $A_1$, $A_2$, $\ldots$ be a decreasing sequence of nonempty compact subsets of $\mathbf{R}^n$. That is, assume that

$A_i \subseteq \mathbf{R}^n$ is compact for all $i$,
$A_i \neq \emptyset$ for all $i$,
$A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots$.

Prove that the intersection $\cap_{i=1}^{\infty} A_i$ is compact and nonempty.

**15.** For two nonempty sets $C$ and $D$ in $\mathbf{R}^n$ we define the *distance* between $C$ and $D$ to be the greatest lower bound of the set of distances between a point of $C$ and a point of $D$. Precisely, we set

$$d(C, D) = \inf\left\{ |x - y| \;\middle|\; x \in C,\ y \in D \right\}.$$

As a notational convention, for $x \in \mathbf{R}^n$ and $D \subseteq \mathbf{R}^n$ we define $d(x, D) = d(\{x\}, D)$.

Prove that for any nonempty set $D \subseteq \mathbf{R}^n$, the function $f : \mathbf{R}^n \to \mathbf{R}$ given by $f(x) = d(x, D)$ is continuous.