MAT 473 **HOMEWORK 3** **Due: Wednesday, 2/12/03**

Homework is due in my mailbox by 4:00. Write neatly, not too small, and not too lightly. You may discuss the problems with other students from class, but you must write your own solutions. **Reread** your proofs **before** copying them out to turn in; I really do mean that you should write (at least) one draft of each solution. If you submit some nonsense in the course of a proof, the whole thing might strike me as not worth reading.

9. Prove that the following function $f : \mathbf{R}^2 \to \mathbf{R}$ is continuously differentiable, and that all second-order partial derivatives of f exist at the origin, but that $D_1 D_2 f(0) \neq D_2 D_1 f(0)$:

$$f(x) = \begin{cases} \dfrac{x_1^3 x_2}{x_1^2 + x_2^2}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

10. Let $f : \mathbf{R}^2 \to \mathbf{R}$ be given by $f(x) = x_1 e^{x_2} + x_2 e^{x_1}$. Find the degree-three Taylor polynomial of f at the origin.

11. Let $f : \mathbf{R}^n \to \mathbf{R}^m$ and $g : \mathbf{R}^m \to \mathbf{R}^\ell$ be twice-differentiable functions.

(i) Write $D_i D_j (g \circ f)_k$ in terms of the partial derivatives of the component functions of g and f.

(ii) Use part (i) to show that

$$(g \circ f)''(x)(v, w) = g''\big(f(x)\big)\big(f'(x)(v), f'(x)(w)\big) + g'\big(f(x)\big)\big(f''(x)(v, w)\big).$$

12. (i) Prove that there exists a nonempty complete metric space (X, d), and a function $f : X \to X$, such that $d\big(f(x), f(y)\big) < d(x, y)$ whenever $x \neq y$, and such that f has no fixed point in X.

(ii) Prove that there exists a contraction mapping f of a nonempty metric space (X, d) such that f has no fixed point in X.