MAT 372 — ADVANCED CALCULUS II — SPRING 2002

Homework 4, due Wednesday, 2/13

Continue reading sections 1-2 of Chapter II.

Problems:

1. Let A_1, A_2, ... be a decreasing sequence of nonempty compact subsets of $\mathbf{R}^n$. That is, assume that

$A_i \subseteq \mathbf{R}^n$ is compact for all i,

$A_i \neq \emptyset$ for all i,

$A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots$.

Prove that the intersection $\cap_{i=1}^{\infty} A_i$ is compact and nonempty.

2. For two nonempty sets C and D in $\mathbf{R}^n$ we define the *distance* between C and D to be the greatest lower bound of the set of distances between a point of C and a point of D. Precisely, we set

$$d(C,D) = \inf\Big\{ |x-y| \;\Big|\; x \in C,\ y \in D \Big\}.$$

As a notational convention, for $x \in \mathbf{R}^n$ and $D \subseteq \mathbf{R}^n$ we define $d(x,D) = d(\{x\},D)$.

Prove that for any nonempty set $D \subseteq \mathbf{R}^n$, the function $f : \mathbf{R}^n \to \mathbf{R}$ given by $f(x) = d(x,D)$ is continuous.