

MAT 371**Homework 10****Due Wednesday, 4/4**

Reading: Sections 5.5 - 5.7, including the notes on section 5.5 and on the natural logarithm and exponential (see the reading assignment for chapter 5 for a list of which portions of the text are optional reading).

Problems

- (1) Page 166, # 9.
- (2) Page 166, # 14 (For odd functions only. Don't use results proved after section 5.3)
- (3) Page 167, # 19 (Don't use Theorem 5.11. Instead, use the following estimate:

if $z, w \in \mathbf{R}$ with $|z|, |w| \geq K > 0$, then

$$\left| \frac{1}{z} - \frac{1}{w} \right| = \left| \frac{z - w}{zw} \right| \leq \frac{1}{K^2} |z - w|.$$

With this, you can estimate $M_i(1/f) - m_i(1/f)$.

- (4) Page 167, # 26.
- (5) Find the limit of the sequence $\{a_n\}_{n=1}^{\infty}$. Prove your answer.

$$a_n = n \left(\frac{1}{1+n^2} + \frac{1}{4+n^2} + \frac{1}{9+n^2} + \cdots + \frac{1}{n^2+n^2} \right).$$

Extra Credit

- (6) Page 167, # 23

Homework 11**Due Wednesday 4/11****Problems**

- (1) Page 167, # 29.
- (2) Page 206, # 1.
- (3) Page 207, # 5.

Extra Credit

- (4) Use the following outline to prove that for any $a > 0$ and $b > 1$,

$$\lim_{x \rightarrow \infty} \frac{x^a}{b^x} = 0.$$

- (i) Prove that $y < e^y$ for $y > 0$. (First prove that $\log x < x$ for $x > 1$.)
- (ii) Prove that $y/e^y < 4/y$ for $y > 0$. (Square both sides of (i), and substitute $y/2$.)
- (iii) Prove the limit in the statement of the problem. The precise formulation of a limit at infinity is: $\lim_{x \rightarrow \infty} f(x) = L$ if

$$(\forall \epsilon > 0) (\exists C > 0) (\forall x \geq C) (|f(x) - L| < \epsilon).$$

- (iv) This shows that every exponential function increases more rapidly than every power of x . Use your work in part (iii) to find how large x must be so that $x^{10}/1.1^x < 1/10$.

Midterm 3 will be given in the Testing Center on Wednesday/Thursday, April 11/12. It will cover chapters 4 - 5.